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AND

DISCUSSIONS

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AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

SETTLEMENT STUDIES OF STRUCTURES IN EGYPT

By Gregory P. Tschebotareff,[1] M. Am. Soc. C. E.

Synopsis

Systematic settlement observations are described, in this paper, of a number of new buildings—public and private—constructed within a radius of several hundred miles. Foundations of every conceivable type were included, and the observations were combined with comparative investigations in the laboratory—all based on the principles of modern soil mechanics.

The organization of these studies is outlined herein, as conducted by the Foundation Soils Research Laboratory of the Egyptian University, at Cairo, Egypt, with the support of the Egyptian Government and the co-operation of local consulting engineers, architects, and contractors. Emphasis is placed on the methods of overcoming the prejudice, ordinarily encountered by practising engineers and architects of all countries, against settlement observations of their structures by independent third parties.

Some of the most important results of studies on eleven of the new structures are presented in graphical form. A comparison is made between modern theories and the results of laboratory tests. The effects of these studies on new trends of local engineering practice are outlined. Conclusions are presented concerning the features considered as being essential for the successful organizations of such studies in any other locality.

Beginning of Settlement Studies in Egypt

Evidence of settlement is to be found on numerous structures erected on the clays and silts of the Nile Valley and Delta. Unforeseen and particularly great difficulties were encountered by the settlements of a large public building erected in Cairo about 1928. Its completion had to be delayed for more than three years until continued observations showed that the rate of settlement had decreased to a reassuring extent. Tests on clay samples and a settlement analysis made by Professor Dr. F. Kögler at his laboratory, in Freiberg,

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Note.—Written comments are invited for immediate publication: to ensure publication, the last discussion should be submitted by **December 15, 1938.**

[1] Asst. Prof. of Civ. Eng., Princeton Univ., Princeton, N. J.; formerly, Research Engr., Foundation Soils Research Laboratory, Egyptian Univ., Cairo, Egypt.

Germany, also showed that the period of consolidation was nearly ended. Some of the settlement observations and other data pertaining to this building (identified herein as Building *T*) are shown subsequently on Figs. 13 and 14. The case attracted attention to the numerous "gaps" in the engineering knowledge concerning the behavior of foundations and to the methods recommended by the new science of Soil Mechanics for the purpose of closing these gaps.

An extensive study of settlement problems was decided upon and the writer was engaged to organize a Foundation Soils Research Laboratory.

Purpose of the Settlement Studies

An investigation was planned of all problems connected with settlements. In particular, the studies undertaken were:

(*a*) To provide the local Engineering Profession with accurate knowledge concerning the final value, the rate and the distribution, in plan, of settlements as observed on actual structures erected on different types of foundations and on different local soil deposits (the factors influencing cracks and other damages to the superstructure were to be established); and,

(*b*) To allow comparisons between the settlement values as actually observed in the field and as forecast on the basis of computations and of laboratory tests; the theoretical methods of analysis were to be corrected and developed by means of such comparisons.

Organization of the Settlement Studies

The following principles guided the organization of the settlement studies:

(1) The studies were classified as a research in the field of public utility. As such, they were financed by the Egyptian Government and the Laboratory was equipped to handle every phase of the investigations—that is, the borings, the extraction and the testing of the samples, and the levelings themselves. The Survey of Egypt assisted in some of the phases of the leveling work.

(2) It was recognized that the usual opposition of practising engineers against settlement observations of their structures might be justified in some cases. This opposition was generally due to fear of damages to the business interests concerned, should settlement observations show that the foundation design had not been fully effective. A number of special measures were necessary, therefore, to safeguard the legitimate business interests involved. These measures were:

(3) A guaranty was given that all data would be kept confidential unless the written approval of those responsible for it was first obtained. The small staff of the Laboratory was made personally responsible for the secrecy of the data in its possession. The staff was authorized, in all cases, to refuse requests for information that were inconsistent with the foregoing provisions, even if the request came officially from the Law Courts.

(4) No charges were to be made in the early stages of the investigations until the degree of accuracy of the methods used had been verified and generally acknowledged.

In this manner the initial reluctance to allow settlement observations was gradually overcome. In December, 1936 (that is, at the end of the first three years of the existence of the Laboratory) twenty-four structures were under observation, of which only three were public buildings and twenty-one were privately owned. The writer understands that this total number was doubled a year later.

Results of the Settlement Studies

Conditions in the Nile Valley and Delta proved to be particularly favorable to the studies. A large number of new buildings were being erected every year. Furthermore, the compressible water-logged clay and silt deposits seldom exceeded a thickness of 30 ft and often averaged only 10 and 15 ft, being underlaid by sand which extended to considerable depths. Since the time of consolidation of a clay layer increases with the square of its thickness, the settlement studies on the relatively thin compressible layers met with in Egypt could give conclusive results much sooner than would be the case in localities with thicker deposits. The results, therefore, may be of more than purely local interest.

Naturally, only an outline of the most pertinent data is possible within the limited space of this paper. This will be realized, if one considers that a report prepared by the writer to cover, fully, the case of only one building (T) consisted of 288 pages of typescript with 72 illustrations.

The liquid and plastic limit, the natural water content, and the laterally unconfined compressive strength test were adopted as routine soil tests for descriptive and classification purposes. The advantage of these tests is that they can be performed rapidly. The liquid (w_e) and plastic (w_p) limit tests illustrate the potential properties of the basic soil material in a disturbed state. The density in the natural state is shown by the natural water content (w_n), the cohesion by the ultimate compressive strength (q_d) and the brittleness or plasticity by the strain at failure.

The subdivision of soils as "clay" or "silt" was made mainly on the basis of the liquid limit values.[2]

The observed settlements and the laboratory test data applying to the first ten buildings studied by the Laboratory are presented graphically on Figs. 1 to 12, which diagrams are intended to be self-explanatory.

Data Concerning the Final Value of Settlements.—It was found that buildings erected on clay soils could settle appreciably, the settlements increasing in direct proportion to the weight of the structure, to the thickness of the stressed clay deposit, and to the value of the compressibility coefficient of the clay type of that deposit. Driving piles into the clay by no means prevented settlements, unless the piles penetrated into the underlying sand. The intensity of stress on the clay layer beneath the pile-bearings and the thickness of that layer appeared to be the governing factors. Thus, the buildings erected on piles, embedded in the clay only, settled: Building T (Figs. 13 and 14), more than 10 in.; Buildings V and X (Figs. 7 and 12), more than 2 in.; and Building VIII

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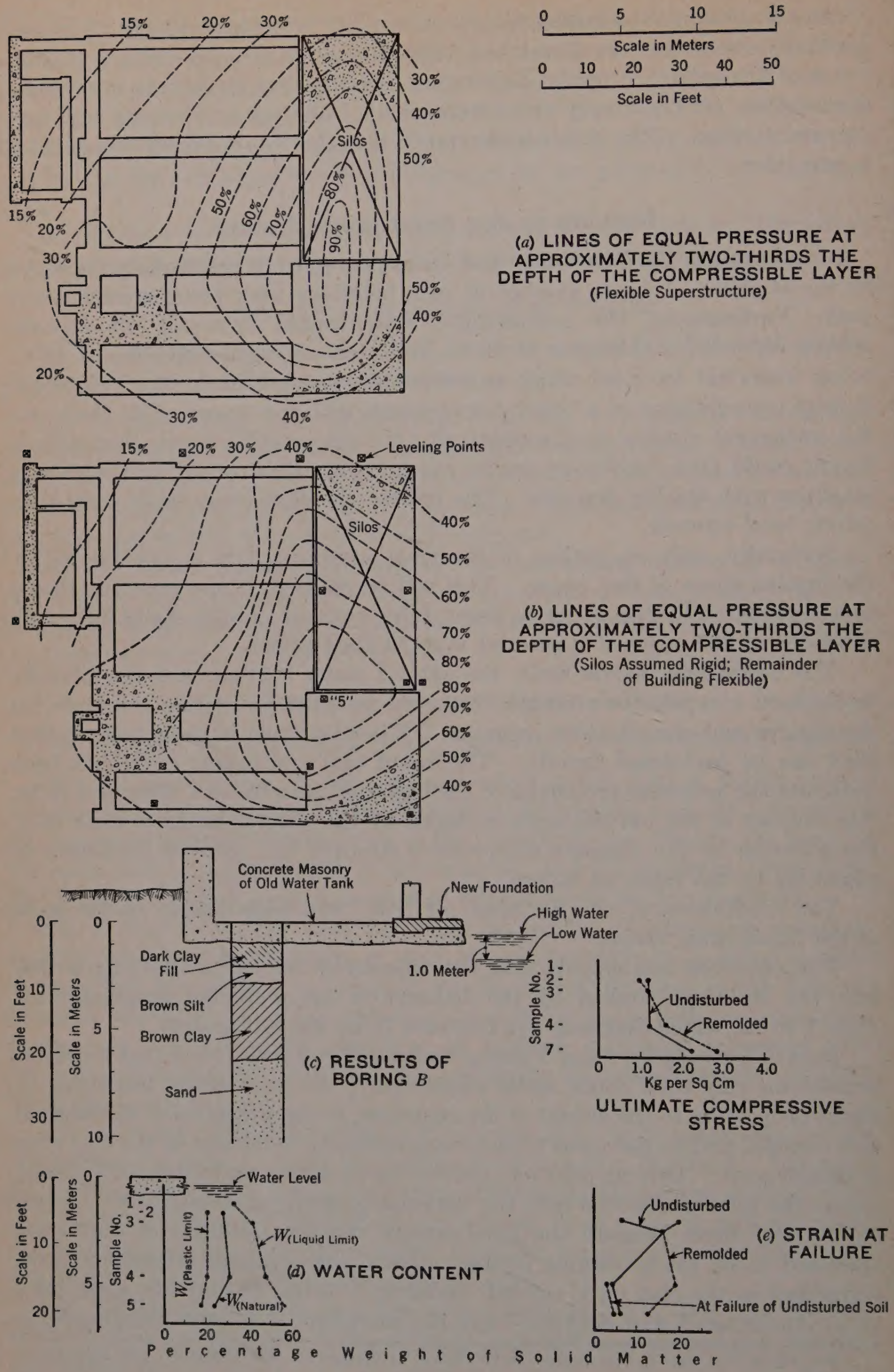
[2] See *Paper C-1*, Vol. I, *Proceedings*, International Conference on Soil Mechanics and Foundation Engineering.



(*a*) LINES OF EQUAL PRESSURE AT APPROXIMATELY TWO-THIRDS THE DEPTH OF THE COMPRESSIBLE LAYER (Flexible Superstructure)

(*b*) LINES OF EQUAL PRESSURE AT APPROXIMATELY TWO-THIRDS THE DEPTH OF THE COMPRESSIBLE LAYER (Silos Assumed Rigid; Remainder of Building Flexible)

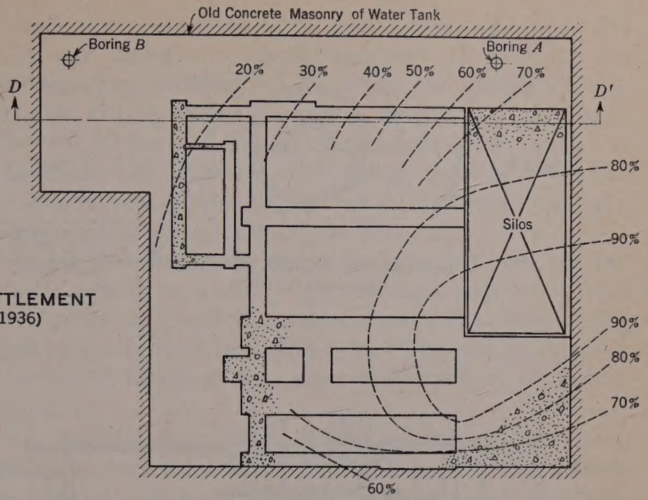
(*c*) RESULTS OF BORING *B*

ULTIMATE COMPRESSIVE STRESS

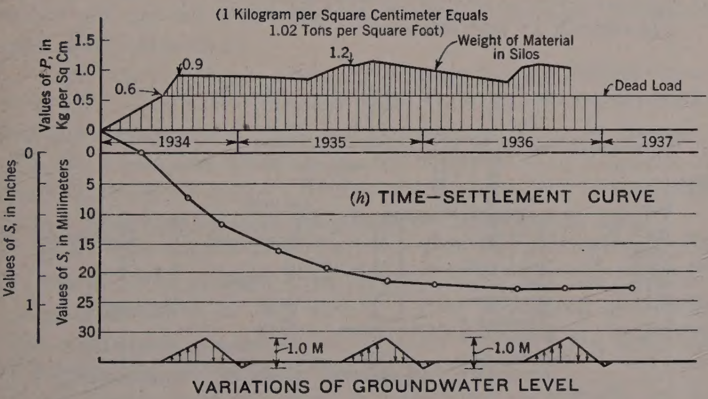
(*d*) WATER CONTENT

(*e*) STRAIN AT FAILURE

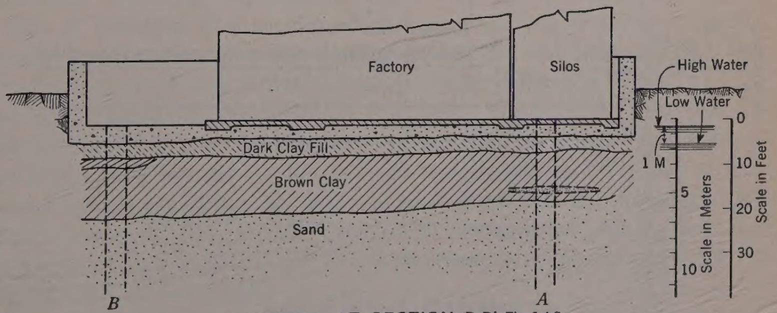
FIG. 1.—



(f) LINES OF EQUAL SETTLEMENT
(Observations to Jan. 1936)

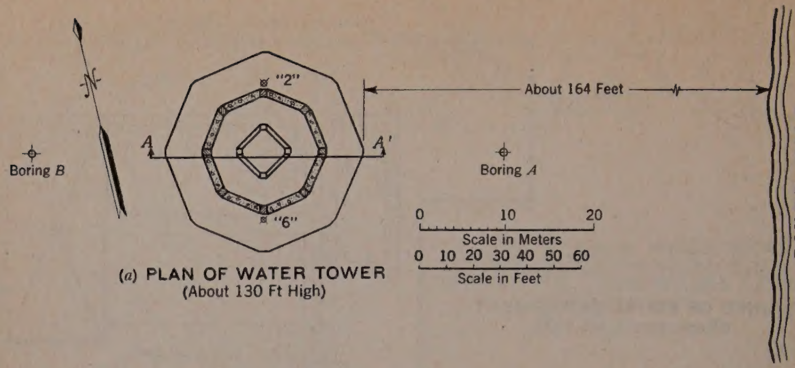


(g) LEVELING POINT "5" MAXIMUM
See Fig 1 (b)

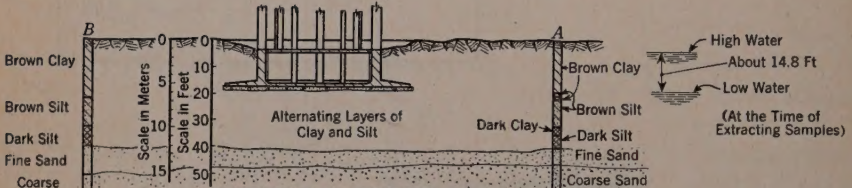


(i) PROFILE AT SECTION D-D', Fig 1 (f)

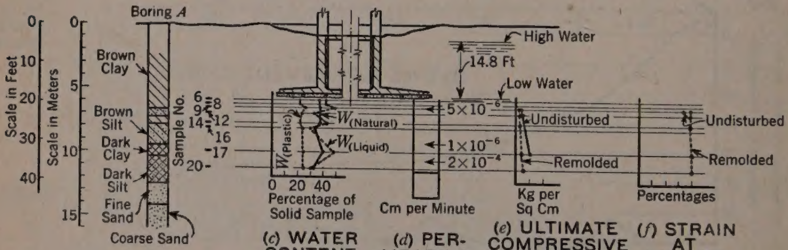
Building I



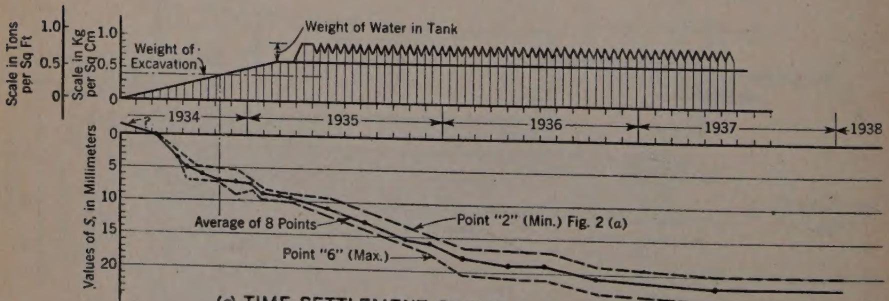
(*a*) PLAN OF WATER TOWER
(About 130 Ft High)



(*b*) PROFILE AT SECTION *A-A'*



(*c*) WATER CONTENT (*d*) PERMEABILITY (*e*) ULTIMATE COMPRESSIVE STRESS (*f*) STRAIN AT FAILURE



(*g*) TIME-SETTLEMENT CURVE

FIG. 2.—BUILDING II

(Figs. 9 and 10), more than 4 in. Building VIII, although only seven stories high, had about 15 ft of clay beneath its pile-bearings and settled five times as much as the heavier ten-story Building VI (in which the settlement was about 1 in., see Fig. 8) with exactly the same system, length, and loading of the piles. In the case of Building VI, the piles penetrated into sand. A five-story structure (Building VII, Fig. 9), on a lighter type of piles just touching the sand surface, had an intermediate value of settlement (slightly greater than 1 in. at one end and almost zero at the other).

Light buildings, weighing less than 1 ton per sq ft of bearing area, erected on spread footings or rafts resting on a stiff variety of local clay of a brown color (Buildings I and III, Figs. 1, 3, and 4), or on compact old silts (Building II, Fig. 2) showed small settlements of less than 1 in. The stiff brown clay and the old silt of the latter two buildings were underlaid by a thick, silty, dark clay layer about 3 ft thick.

When the stiff brown clay was underlaid by a thicker layer of a local clay, about 10 ft thick, of the same dark color but more plastic, equally light structures on spread footings or rafts settled substantially: The three-story structure (Building IV, Figs. 5 and 6) settled more than 3 in. and a light four-story villa (Building IX, Fig. 11) settled more than 6 in. In this latter case a recent silt deposit overlying the dark clay appears to have contributed to part of the settlement.

Comparison Between Load-Test Results and the Settlements of the Full-Sized Structures.—No useful relation could be established between load-test results and the settlements of full-sized structures. Thus, in the case of Building II (Fig. 2), a load test made on an area of about 10 sq ft did not show any measurable settlement under a load of 1 ton per sq ft (accuracy of reading about $\frac{1}{32}$ in.), under which load the structure settled about 1 in.

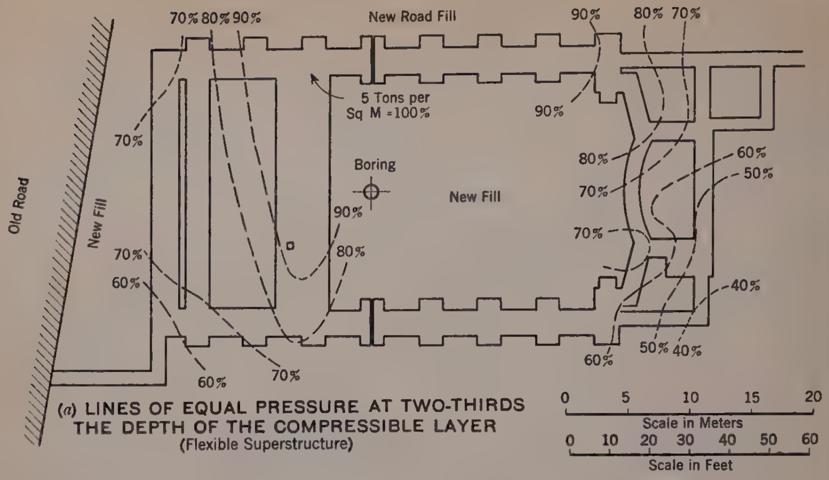
In another similar case (Building IX), the dry clay soil at the level of the foundation was so hard that no impression was left by the heel of a man spinning round on it. Obviously, a load test would have been useless under such conditions, yet the entire structure settled 6 in., due to the compression of deeper lying layers below subsoil water level.

Load tests on such layers in bore-holes gave much too high settlement values due to the inevitable loosening of the wet exposed soil surface. Under similar conditions load tests, therefore, appear inadvisable for any purposes.

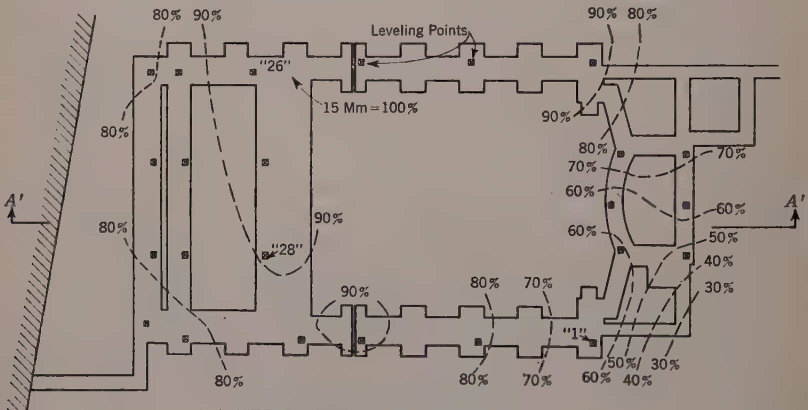
In the case of Building *T*, test-piles observed for periods of several months until complete stabilization showed settlements about one-twentieth of the actual settlement of the entire structure under the same load per pile.

All piles of the buildings described in this paper are concrete piles cast in place. The steel tube driven into the ground is withdrawn and the concrete rammed with a heavy tamper. This tamper weighs about 1 500 lb for the type of piles used in Buildings V, VII, X and *T* (the usual working load allowed locally on such piles varies from 30 to 45 tons per pile). For the type of piles used in Buildings VI and VIII the tamper weighs about 4 500 lb (usual working load, 60 to 80 tons per pile).

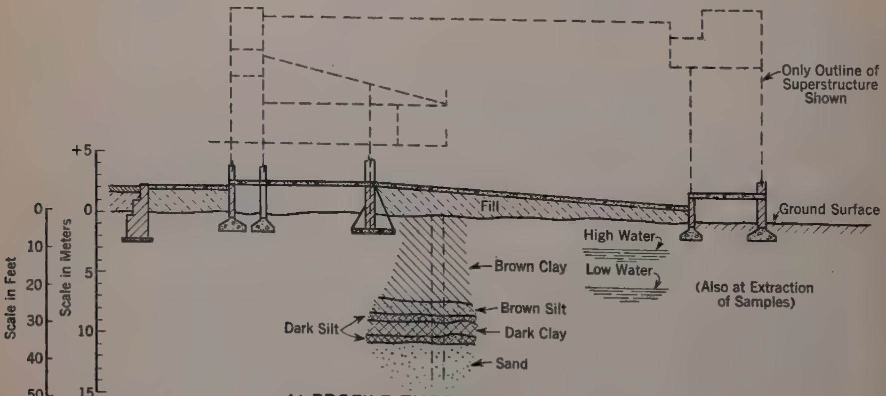
No load tests had been made for the other structures by the engineers in charge, since it was well known locally that, under the unit loads used in the



(*a*) LINES OF EQUAL PRESSURE AT TWO-THIRDS THE DEPTH OF THE COMPRESSIBLE LAYER
(Flexible Superstructure)



(*b*) LINES OF EQUAL SETTLEMENT
(Observations to Feb. 1936)



(*c*) PROFILE THROUGH *A'-A'*

FIG. 3.—BUILDING III

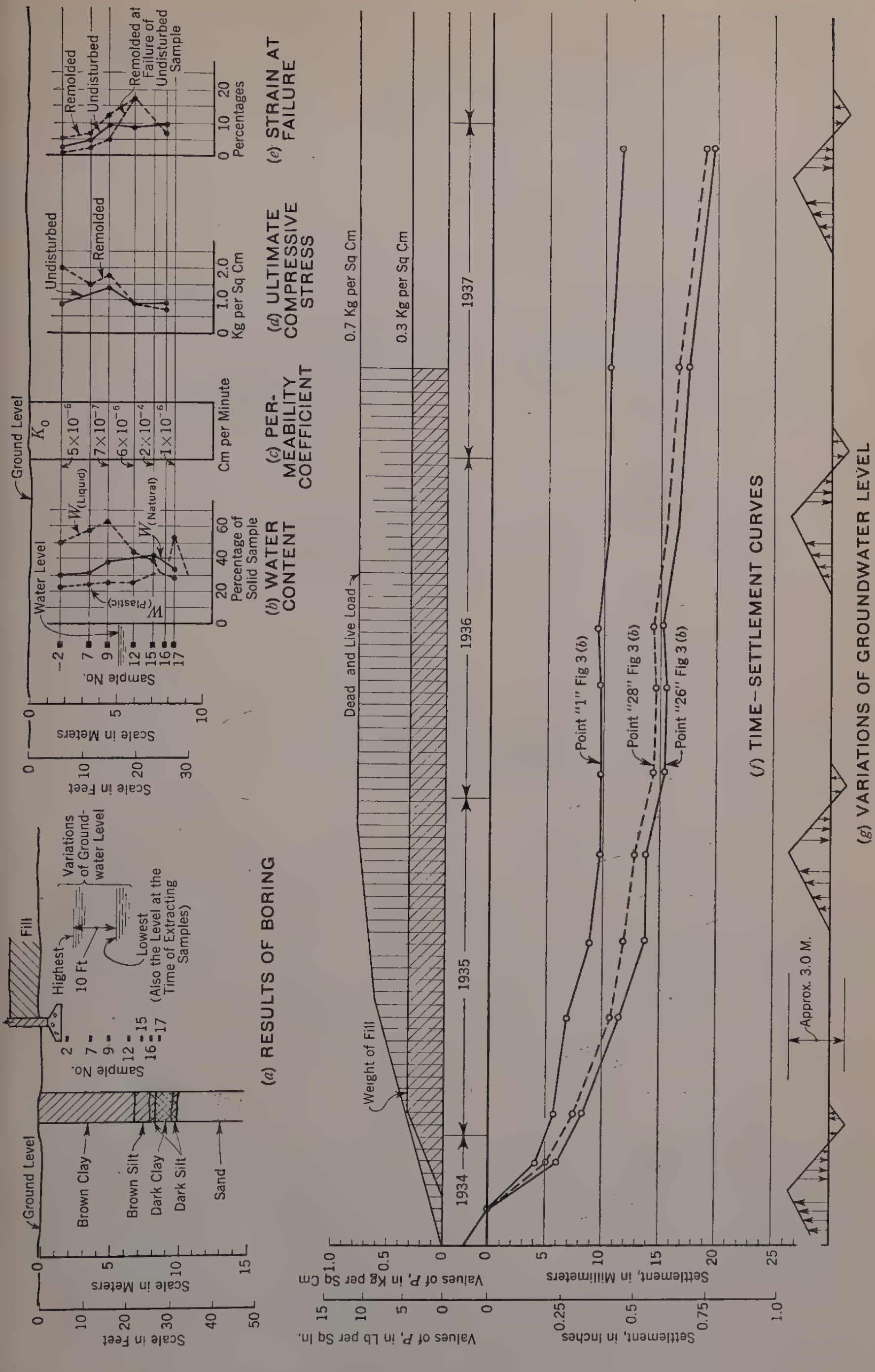
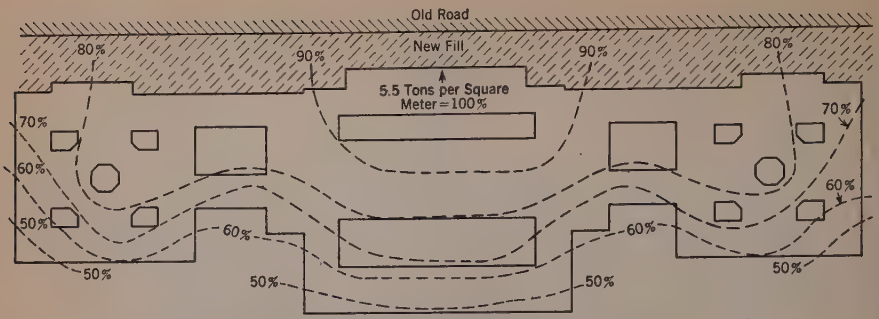
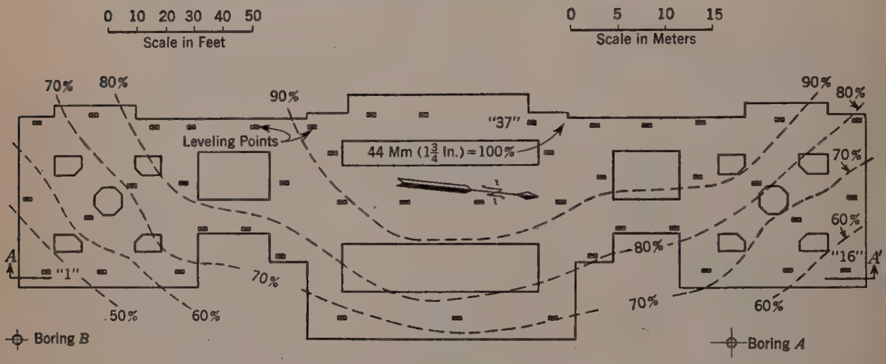


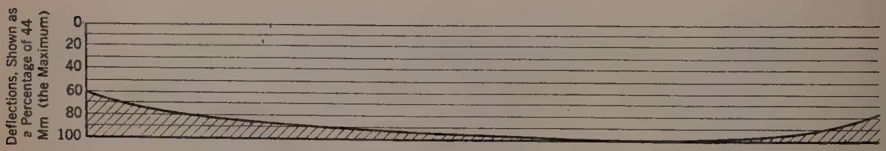
FIG. 4.—BUILDING III



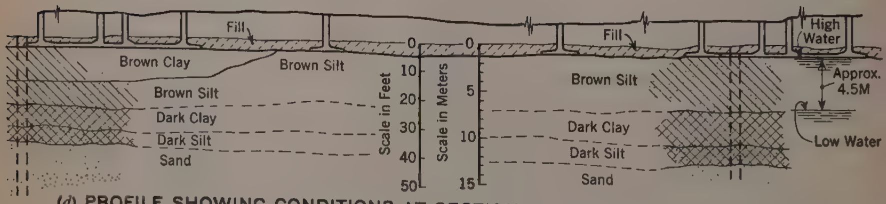
(*a*) LINES OF EQUAL PRESSURE AT APPROXIMATELY TWO-THIRDS THE DEPTH OF THE COMPRESSIBLE LAYER
(Flexible Superstructure)



(*b*) LINES OF EQUAL SETTLEMENTS
(Observations to Jan. 1936)

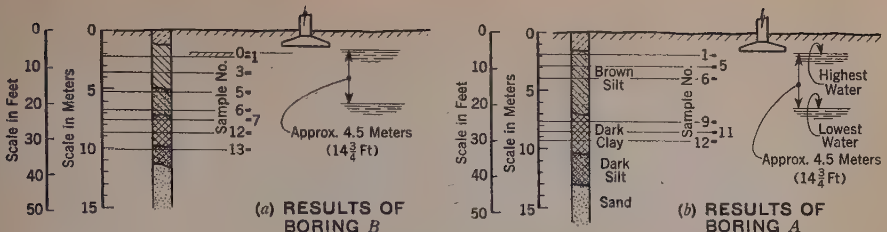


(*c*) OBSERVED DEFLECTIONS OF WEST WALL



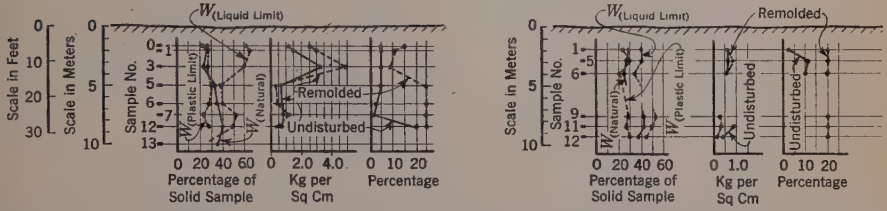
(*d*) PROFILE SHOWING CONDITIONS AT SECTION *A-A'*, A THREE-STORY BUILDING

Fig. 5.—Building IV



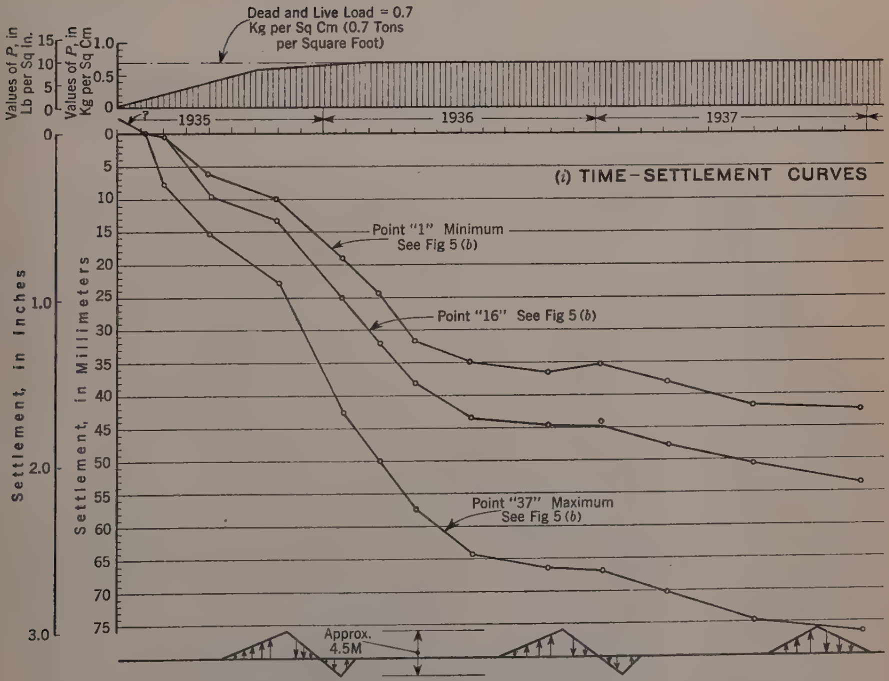
(a) RESULTS OF BORING B

(b) RESULTS OF BORING A



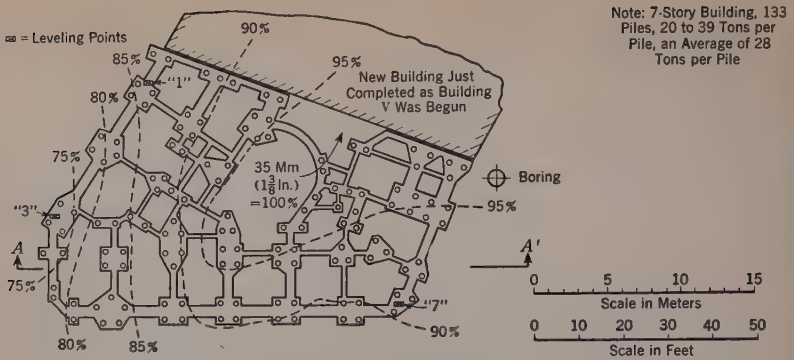
(c) WATER CONTENT (d) ULTIMATE COMPRESSIVE STRESS (e) STRAIN AT FAILURE

(f) WATER CONTENT (g) ULTIMATE COMPRESSIVE STRESS (h) STRAIN AT FAILURE

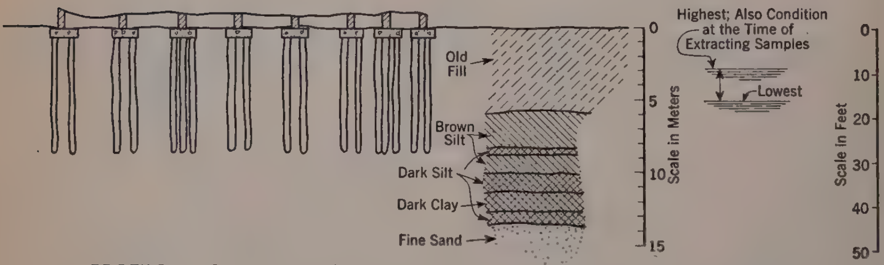


(j) VARIATIONS IN SUBSURFACE WATER LEVEL

Fig. 6.—Building IV

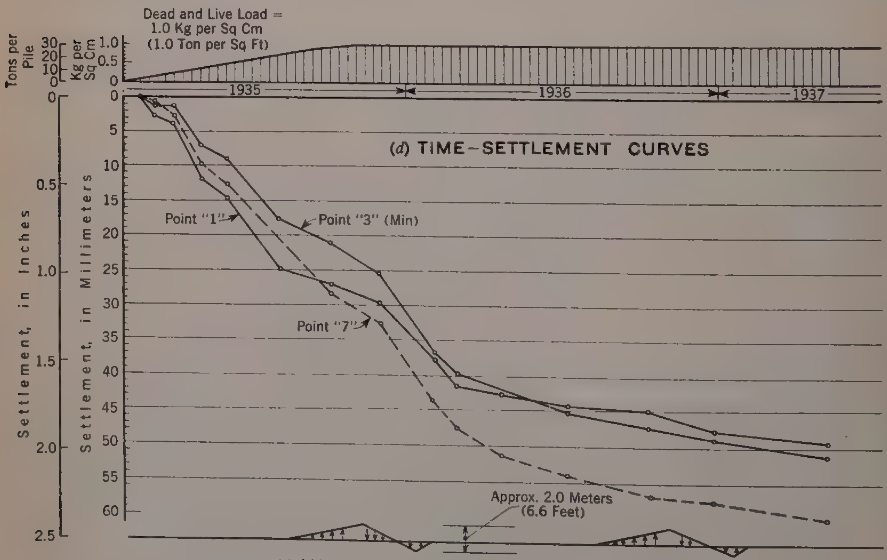

▭ = Leveling Points
90%
85%
"1"
95%
New Building Just Completed as Building V Was Begun
80%
75%
35 Mm ($1\frac{3}{8}$ In.) =100%
"3"
Boring
95%
A
A′
75%
"7"
90%
80%
85%
0 5 10 15
Scale in Meters
0 10 20 30 40 50
Scale in Feet
Note: 7-Story Building, 133 Piles, 20 to 39 Tons per Pile, an Average of 28 Tons per Pile

(*a*) LINES OF EQUAL SETTLEMENTS
(Observations to Dec. 1935)



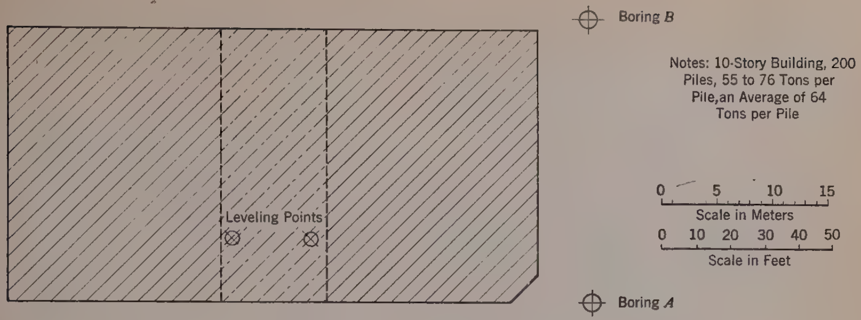
(*b*) PROFILE OF SECTION *A-A′*

(*c*) WATER LEVELS

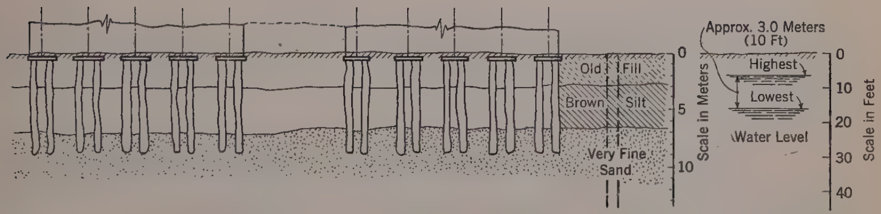


(*e*) VARIATIONS OF SUBSOIL WATER LEVEL

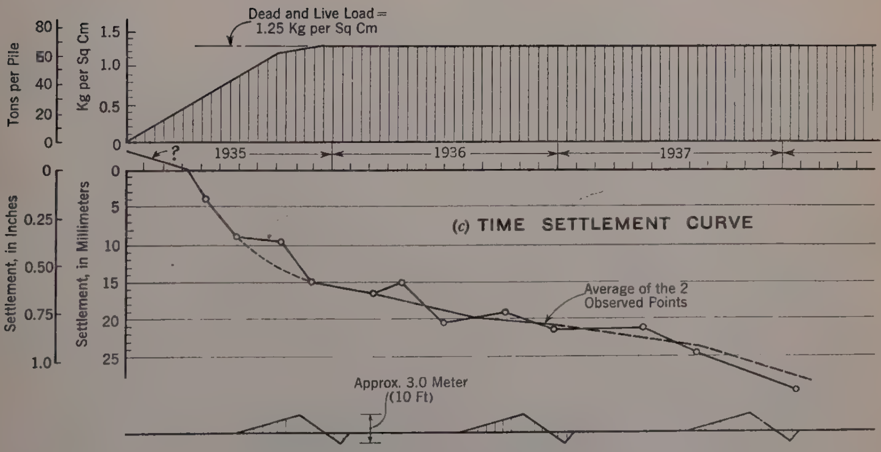
FIG. 7.—BUILDING V


Boring B
Notes: 10-Story Building, 200 Piles, 55 to 76 Tons per Pile, an Average of 64 Tons per Pile
0 5 10 15
Scale in Meters
0 10 20 30 40 50
Scale in Feet
Leveling Points
Boring A

(*a*) PLAN OF TEN-STORY BUILDING

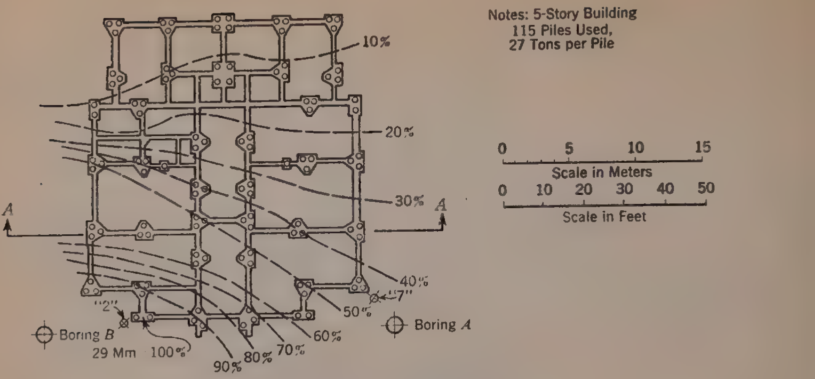


(*b*) PROFILE SHOWING SUBSOIL CONDITIONS

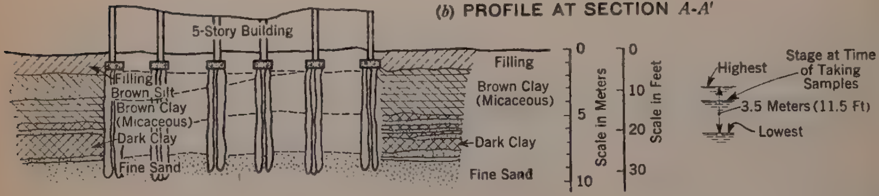


(*d*) VARIATION OF GROUNDWATER LEVEL

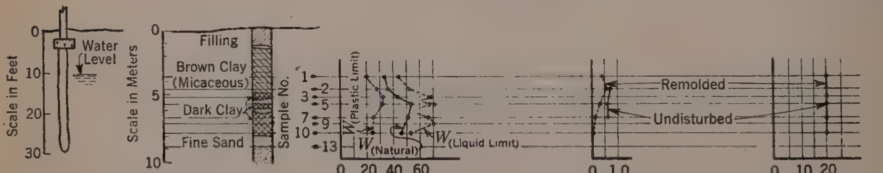
Fig. 8.—Building VI



(a) LINES OF EQUAL SETTLEMENTS
(Observations to Jan. 1936)



(b) PROFILE AT SECTION A-A′

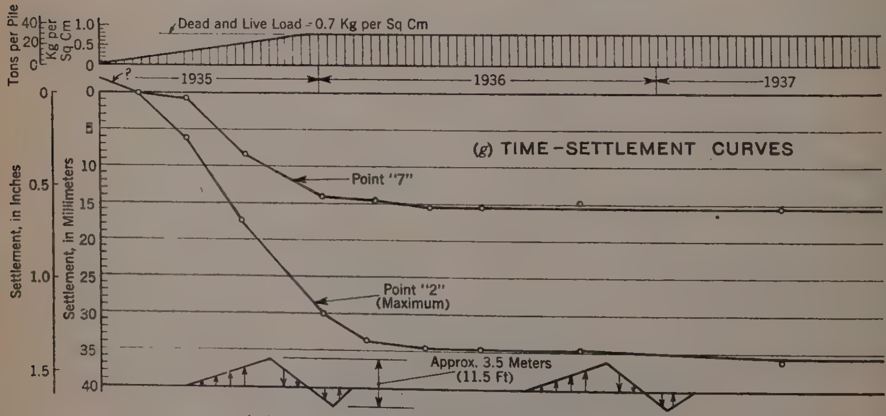


(c) BORING A

Percentage of Solid Sample
(d) WATER CONTENT

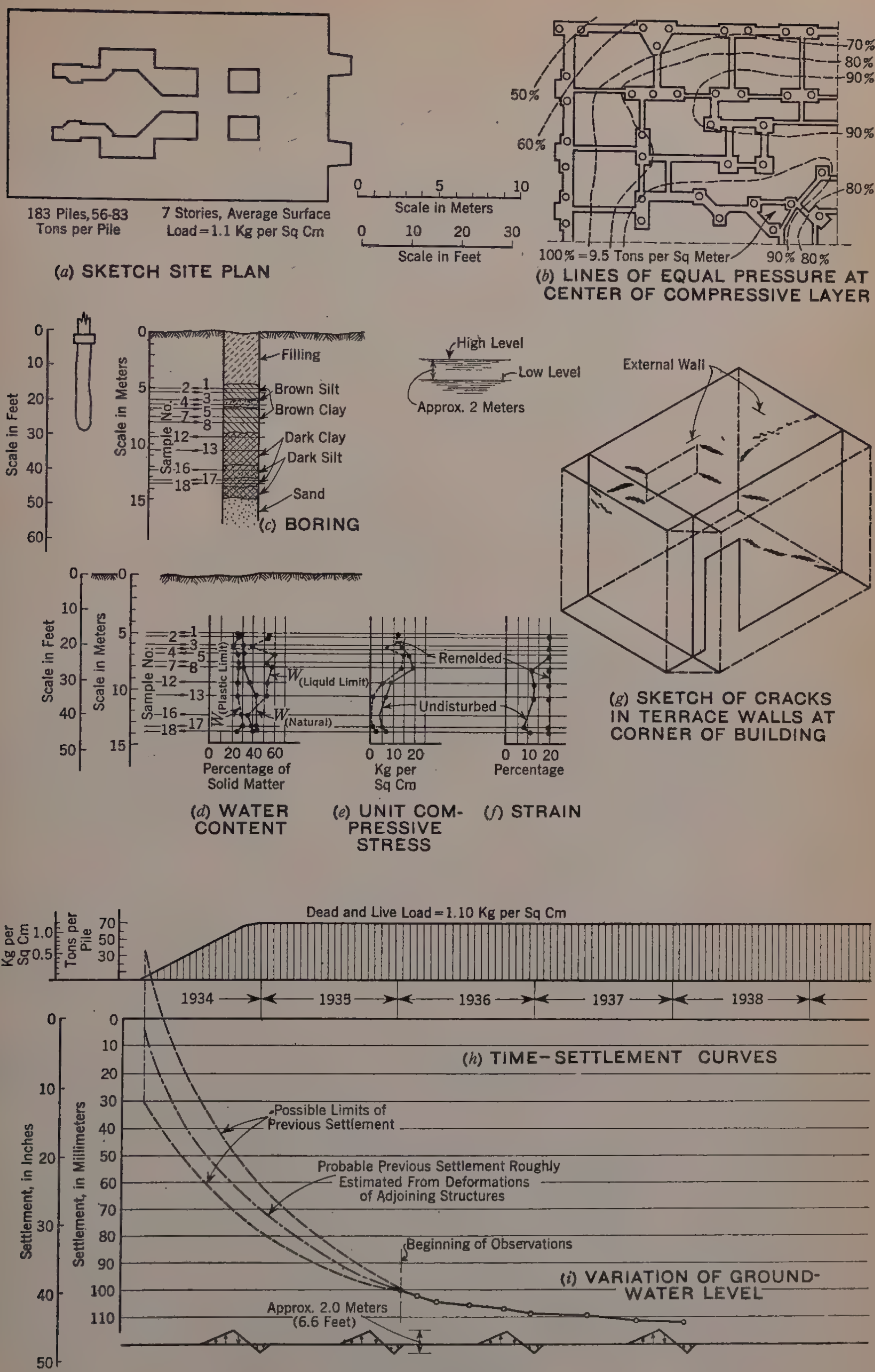
Kg per Sq Cm
(e) ULTIMATE COMPRESSIVE STRESS

Percentage
(f) STRAIN AT FAILURE



(h) VARIATIONS OF SUBSOIL WATER LEVELS

FIG. 9.—BUILDING VII



(a) SKETCH SITE PLAN

(b) LINES OF EQUAL PRESSURE AT CENTER OF COMPRESSIVE LAYER

(c) BORING

(d) WATER CONTENT

(e) UNIT COMPRESSIVE STRESS

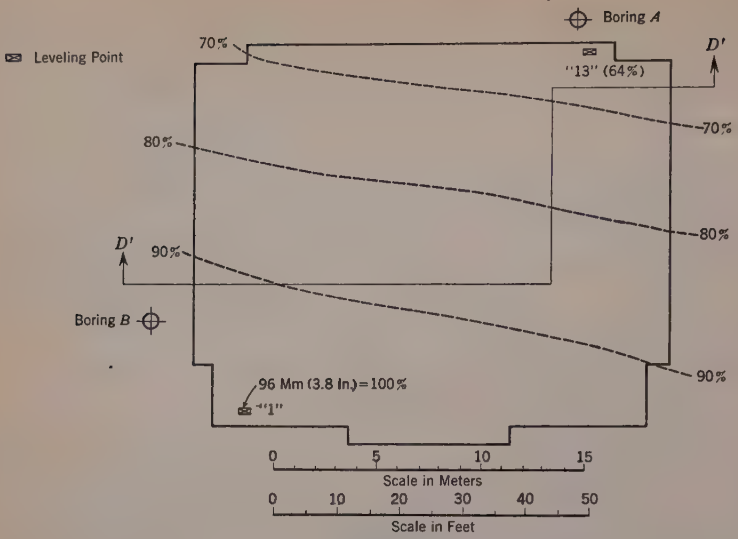
(f) STRAIN

(g) SKETCH OF CRACKS IN TERRACE WALLS AT CORNER OF BUILDING

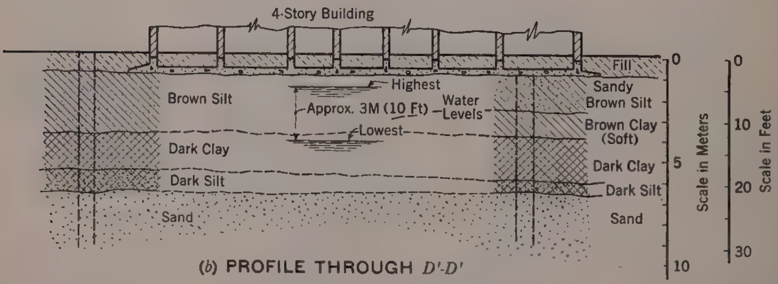
(h) TIME-SETTLEMENT CURVES

(i) VARIATION OF GROUND-WATER LEVEL

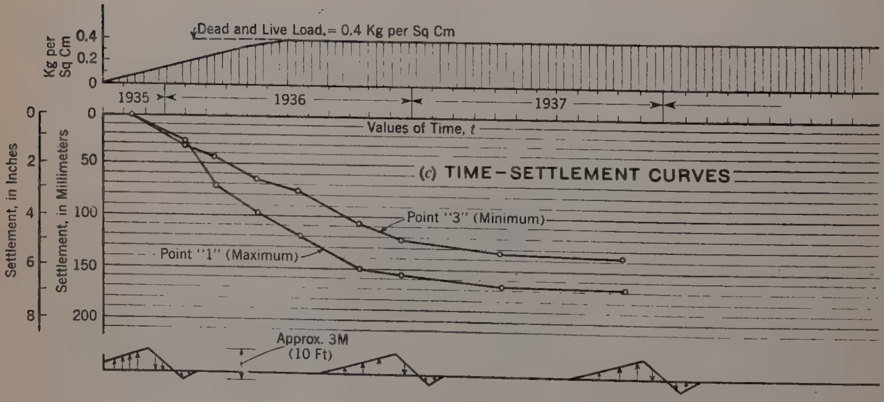
FIG. 10.—BUILDING VIII



(*a*) PLAN AND LINES OF OBSERVED EQUAL SETTLEMENTS
(To May 1936)

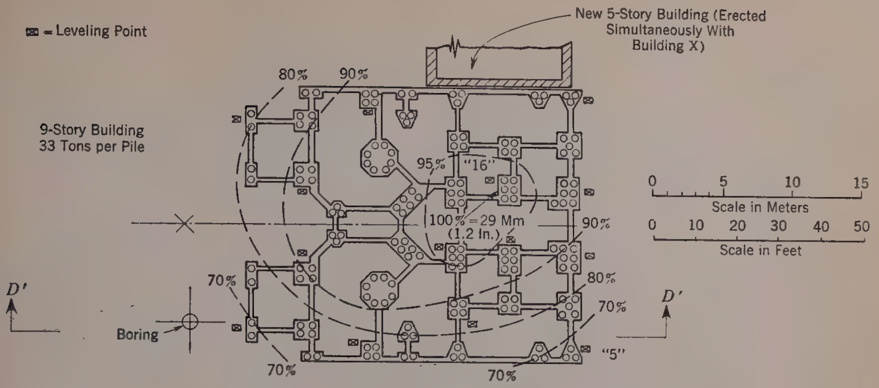


(*b*) PROFILE THROUGH *D'-D'*

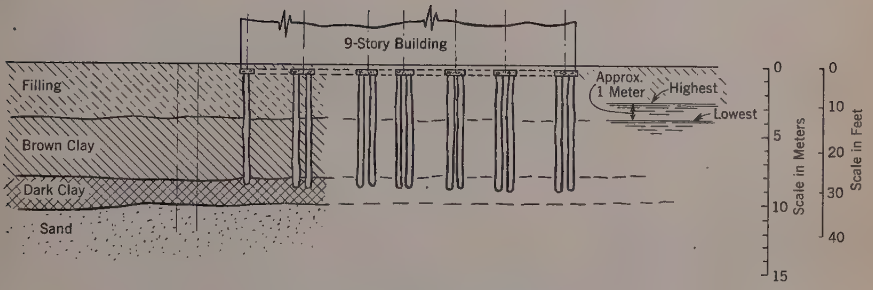


(*d*) VARIATIONS OF SUBSOIL WATER LEVELS

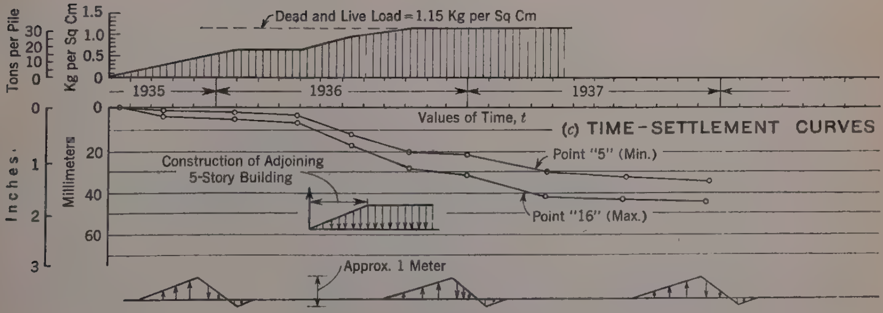
FIG. 11.—BUILDING IX



(a) LINES OF OBSERVED EQUAL SETTLEMENTS
(Observations to Dec. 1936)

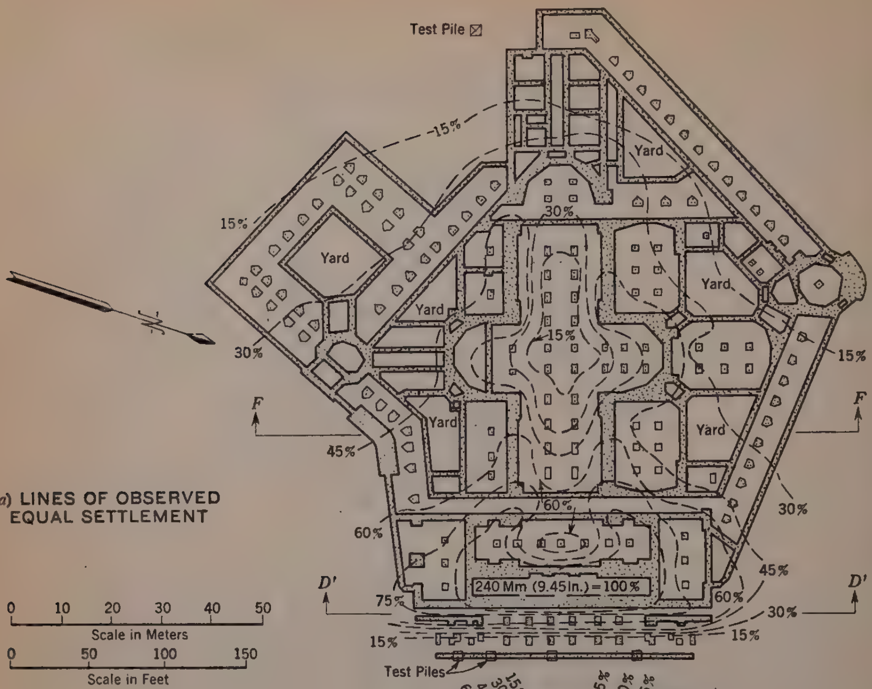


(b) PROFILE THROUGH SECTION D'-D'

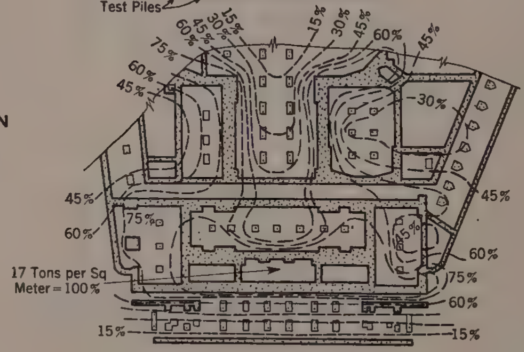


(d) VARIATIONS OF GROUNDWATER LEVEL

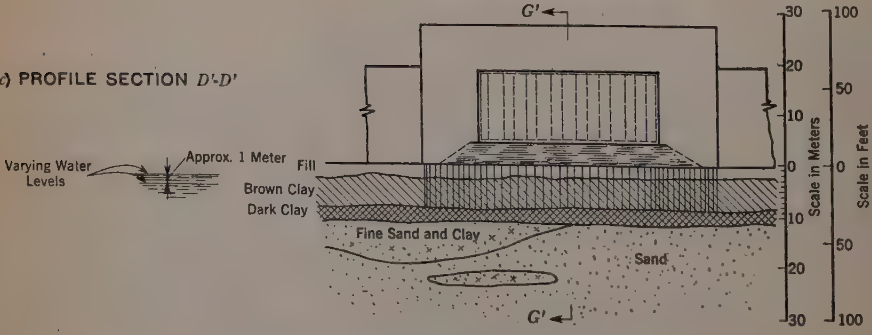
FIG. 12.—BUILDING X



(*a*) LINES OF OBSERVED EQUAL SETTLEMENT

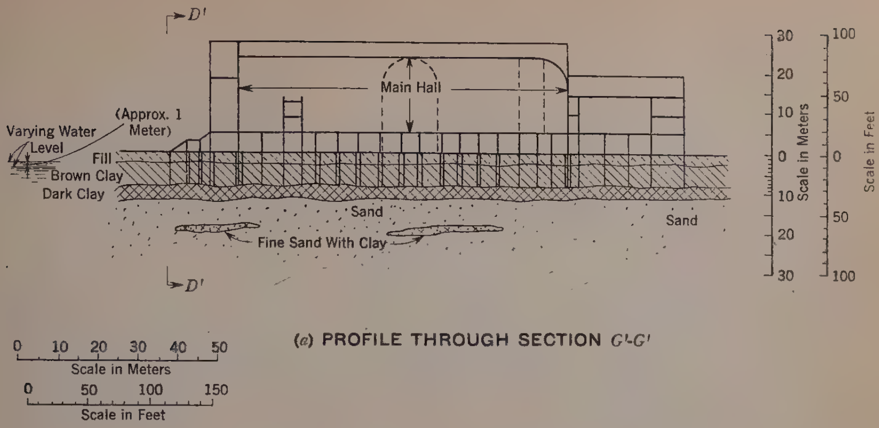


(*b*) PART PLAN OF FOUNDATION SHOWING COMPUTED LINES OF EQUAL PRESSURE AT 10-METERS DEPTH (Center of Compressible Layer)

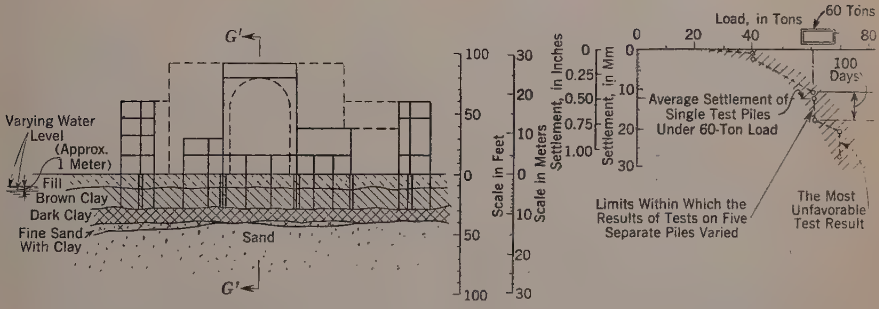


(*c*) PROFILE SECTION *D'-D'*

FIG. 13.—BUILDING *T*

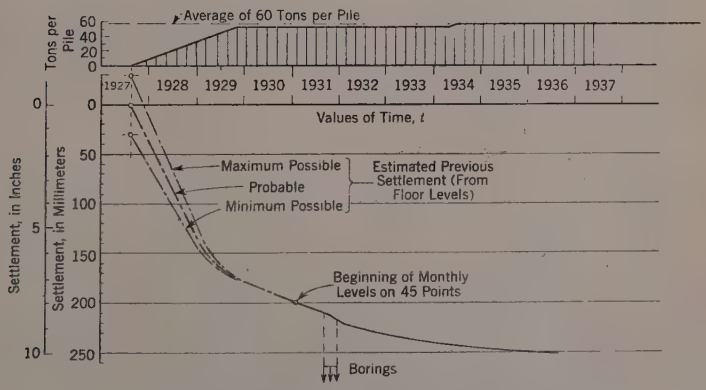


(a) PROFILE THROUGH SECTION G'-G'



(b) PROFILE THROUGH SECTION F-F

(c) PILE LOAD TESTS



(d) TIME-SETTLEMENT CURVE OF ZONE OF GREATEST SETTLEMENT—(EAST ENTRY)

FIG. 14.—BUILDING T

designs, single test-piles showed negligible settlements even when embedded in clay only.

No Decrease of Settlement Was Observed as a Result of Excavation.—As recently as 1933 excavation of soil was generally advocated as a useful means of decreasing settlements, which were not supposed to begin until the weight of the superstructure had exceeded that of the soil removed. Actually, the opposite was observed (see Building II, Fig. 2). Settlements were registered immediately after loads were applied to the soil. This was probably caused by the preceding elastic swelling of the underlying moist soil in the extensive period between the end of the excavation and the beginning of construction.

Relation Between Settlements in Clays as Actually Observed and as Forecast on the Basis of Laboratory Tests.—Forecasts based on consolidation tests made on the stiff and easily swelling brown clays from the first three buildings studied gave two or three times greater settlement values than those which were actually observed. The writer then took samples of the clay to the laboratory of Charles Terzaghi, M. Am. Soc. C. E., at Vienna, Austria, and the results of the tests made there fully agreed with his own. It became quite obvious that the discrepancy could not be attributed to an individually faulty procedure, but that it was due to inadequacies of the usual standard method of testing.

Subsequently, the writer proved that the discrepancy was due to an elastic swelling of the brown clay samples during and after their removal from their position in the natural deposit.[3] An agreement between the observed coefficients and those derived by laboratory tests was reached by loading the stiff brown clay sample in the consolidation device up to the value of the natural pre-consolidation load; then unloading to the pressure corresponding to the weight of the overlying layers, loading again, and only then using the settlement coefficients from this latter branch of the compressibility pressure curve. (This pre-consolidation load, determined separately, is comparatively high due to repeated drying and swelling of the soil deposits after, and during, the flood seasons.)[4]

To compare the observed coefficients and those derived from observations on laboratory-test settlements, the writer introduced a new compression coefficient, X, which gave directly the unit compression of the layer, or of the sample, under an average unit stress:

$$X = \frac{a}{1 + e'} \times 1\,000 = \frac{s}{\Delta p \times d'} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which: a = compressibility coefficient, in square centimeters per gram, as determined by the standard testing procedure from the voids ratio-pressure curves; e' = initial voids ratio; s = observed settlement of layer or compression of test sample; Δp = increment of the average pressure on the layer, or on the sample; and, d' = initial thickness of the layer or of the sample.

The use of Equation (1) permitted the determination of the compression coefficient, X, at any stage of loading during the laboratory compressibility test with not more than 3% or 4% error and without having to wait for the end of the test and the determination of the voids ratio of the sample.

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[3] *Proceedings*, International Conference on Soil Mechanics and Foundation Engineering, *Paper D-1*, Vol. I, 1936.

[4] *Proceedings*, Am. Soc. C. E., October, 1937, p. 1621.

Subsequent observations on buildings underlaid by the non-swelling plastic and weaker dark clays showed that a fairly good agreement with the observed values was obtained for these clays by the usual standard test procedure; that is, by taking the settlement coefficients from the voids ratio-pressure curve for the first loading after the test pressure had exceeded the weight of the over-burden.

Therefore, the method of taking a sample, combined with the potential capacity of a soil to swell, or to fall apart, appears to be among the factors primarily affecting the accuracy of settlement forecasts and the advisable method for the interpretation of test results. Another important factor is the original stiffness of the material because the same disturbance while removing a sample will cause a relatively greater error in the settlement forecast for a stiff clay than for a weak one.

This is a probable explanation of why most of the earlier laboratory settlement analyses gave such good agreement with the observed field values; the first investigations were made where some foundation trouble had occurred, which may be taken as an indication in itself of the presence of weak soil material. The entire question, however, takes on a totally different aspect if, as a matter of routine, one is called upon to make settlement forecasts for varying types of soil on which no previous settlement observations have been made. One often hears about the necessity of "experience" in the interpretation of soil test results. Generally, this means the number of tests and analyses performed within the four walls of a laboratory. The writer wishes to emphasize that such experience is of little value unless it is accompanied by numerous comparisons with field observations on full-scale structures. Furthermore, the experience derived from such comparisons in one locality cannot always be applied with equal success in another locality with a different geological history of its soil deposits. Hence, it follows that settlement should be studied in all localities if indications of local and general practical value are to be obtained.

The following are some average values of the compression coefficient, X, for Egyptian soils, as approximately determined from the observations on actual structures:

Material	Values of X
Plastic dark clay	0.070 to 0.030
Loosely deposited recent silt	0.070 to 0.030
Stiff brown clay	0.007
Compact old silt	0.003

These values were determined for metric linear dimensions and for stresses expressed in kilograms per square centimeter (equivalent to about 2 044 lb per sq ft); that is, $X = 0.010$, would mean that a layer 1 m thick would compress 1 cm (0.4 in.) under an average vertical unit stress of 1 kg per sq cm (14.5 lb per sq in.) throughout its entire thickness. Since this stress is approximately equal to a stress of 1 ton per sq ft, the foregoing values of X remain unchanged if the thickness of the layer is expressed in feet and the stress in tons per square foot. In all cases the underlying sand deposits were assumed to be relatively incompressible.

Efficiency of Devices for Taking "Undisturbed Samplers."—The so-called "undisturbed samplers" used had a cutting-edge of a slightly smaller diameter than the diameter of the sample-containing tube. They were similar to the so-called M.I.T. type and were found to yield reliable information as to the composition, the structure, and the approximate stiffness of the layers encountered. They preserved, intact, and revealed, important details of the layer formation none of which could be noticed if and when ordinary augers were used; the latter disturbed the structure of the soil completely and mixed its various constituents.

Thus, the "undisturbed samplers" were found to be quite efficient for purposes of soil identification and to be definitely preferable in this respect to the augers and to other more primitive methods of soil exploration. However, samplers of this type could not be considered as giving entirely undisturbed samples to the extent required for laboratory compressibility investigations, because they allowed the expansion and the swelling of the soil within the sample-containing tube. This swelling caused errors in the settlement forecasts of more than 100% and, sometimes, even 200 per cent. The narrower cutter appears to have been originally introduced by the designers of this type of sampler for the purpose of decreasing friction along the walls of the container tube, this friction having been considered as the source of greatest possible disturbance. In view of the facts observed, it appears likely that the unfavorable effects of this lateral friction have been greatly over-estimated, especially if compared to the effects of the dislocation of the structure of the soil due to lateral expansion in the container following the use of narrower cutters.

In the writer's opinion, constant-diameter samplers, at least 4 in. wide, should be tried for soils in which lateral expansion is likely to cause considerable disturbance—for instance, for pre-compressed clays of a swelling type. In such clays a state of considerable horizontal stress is likely to exist,[4] and an uninterrupted lateral confinement becomes essential for all stages during the transfer of the sample from its position in the natural deposit to the compressibility testing apparatus in the laboratory. Friction along the walls of the container is likely to be detrimental only to clays the structure of which is broken by remolding.

Not All Clays Are Unfavorably Affected by Remolding.—Compressive strength tests on numerous clay samples, both in the undisturbed and in the remolded states, showed that the dark clay samples were very unfavorably affected by remolding. This was not the case for the stiffer brown clay samples, which were only slightly affected, or not at all. The types of brown clay that were likely to swell more appeared to be least affected by remolding. The fundamental causes of this difference in behavior are not quite clear and require investigation.

Estimation of the Compressibility of Silts.—Comparison of the settlement records of Buildings II (Fig. 2), III and IV on one hand and of Building IX on the other (Fig. 11) showed that the compressibility of loose, recently deposited silts could be more than ten times as great as that of compact old silts and could produce appreciable settlements (Building IX, Fig. 11). At the same time, there are no simple means of establishing, rapidly and beforehand, whether one is dealing with a compact, or with a loose, silt. Such silts with little cohesion cannot be extracted from bore-holes below ground-water level in a really undisturbed state as required for laboratory testing. Furthermore, there is very little hope that this will ever be attained by any new system of samplers.

Nevertheless, for practical purposes, it is essential to be able to estimate the degree of compressibility of silts on new building sites. It seems to the writer that the only method which may give satisfactory results is the use of small cones slowly jacked into and through the silt or sand deposit. The pressure applied in relation to the penetration registered gives comparative empirical coefficients of the density and of the compressibility of the soil penetrated. Such methods were tried first by Professor Terzaghi in the United States and then by Dutch engineers in Holland.[5] They appear to be in need of correlation to observations and of further development, but they form what seems to be a promising innovation.

Rate of Settlement.—The rate of settlement observed was found to be in good agreement with the theoretical rate based on the permeability of the water-logged deposits stressed by the weight of the buildings when they were founded on mat or strip-footing foundations. The settlements of Building II (Fig. 2) continued for a much longer time than could be expected from the permeability tests. This continued settlement appears to be due to the constant variations of 20% of the total load twice weekly before and after the refilling of the water tank.

The settlements of light structures founded on piles resting on sand (Building VII) stopped almost immediately after construction. On the other hand, the settlements of similar but heavier structures (Building VI) continued long afterward. The causes of this latter phenomenon are not clear but may be attributed to the effect of gradual consolidation of the overlying clay and silt layers and the resulting decrease of the frictional support they temporarily provided, combined with lateral and upward flow of the stressed sand beneath their bearings.

Ground-Water Level.—The effects of ground-water level variations were found to be much less important than was generally believed. These variations depend on the Nile flood which occurs yearly at almost exactly the same periods and with very much the same amplitude. The variation in the ground-water level ranges from 1 ft to about 18 ft, according to the distance of the site from the river and to the permeability of the adjoining soil layers. Before the settlement studies were undertaken, many engineers believed that these yearly variations should be held responsible for a dislocation of the soli structure which then caused any of the settlements noticed. An examination of the time-settlement curves of the observed structures shows that this is not the case.

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[5] *Proceedings*, International Conference on Soil Mechanics and Foundation Engineering. Rept. from the Delft Laboratory, *Paper I–I*, Vol. I, p. 181.

A pronounced effect on the time-settlement curve was observed only when the rise of the ground-water level coincided with the period of construction and when the seat of settlement was located in a deep, dark, clay layer (Buildings IV and V, Figs. 5, 6, and 7). This may be explained as follows: A rise of the ground-water level decreases, by the amount of buoyancy, the pressure component on the deep, dark clay due to the weight of the overlying layers within the height of the water rise. This dark clay has been fully consolidated under the maximum pressure from the overlying soil corresponding to the lowest ground-water level. When the water rises at the same time as the building is constructed, not all, if any, of the newly applied load of the superstructure creates stresses in the dark clay layer in excess of the previous maximum. This newly applied surface load is compensated by the decrease in the weight of the newly saturated soil layers. The rate of settlement decreases and the time-settlement curve flattens out. When, subsequently, the ground-water level falls, all the surface load of the structure erected during the period of the water rise is suddenly applied to the deeper layer of dark clay. It creates stresses in excess of the preceding maximum and a sudden and rapid increase in the rate of settlement results (Buildings IV and V, Figs. 5, 6 and 7). The same process may be repeated on a much smaller scale in succeeding years until consolidation under the effect of the load of the structure is completed.

This shows that the variations in ground-water level, in soils of the type studied, only slow down the rate of settlement during the rise of the water level and accelerate it during its fall. The final value of the settlements does not appear to be affected.

Lines of Equal Settlement.—The distribution of the settlements in plan seemed to be governed in most cases by the intensity of stress at different points of the same planes within the clay layers, which seemed, therefore, to have a horizontally uniform degree of compressibility. Lines of equal pressure, computed from the Boussinesq formulas at about two-thirds the depth of the compressible layer, or at its center, showed an agreement of shape and of relative magnitude which had to be considered as fairly good if the distributive action of the semi-rigid superstructure is taken into account as demonstrated by Buildings I (Fig. 1), III (Figs. 3 and 4), IV (Figs. 5 and 6), V (Fig. 7), X (Fig. 12), and *T* (Figs. 13 and 14). The inclination of cracks in Building VIII (Fig. 10) furnished evidence in the same direction; the horizontally uniform compressibility of the soil was also evidenced by the uniform settlement of the perfectly rigid water tower in Building II (Fig. 2), which had only a slight tilt in the direction of the prevailing winds.

The remaining two structures, Buildings VII and IX, in Figs. 9 and 11 (Building VI could be observed only on two points) unexpectedly showed irregular settlements which obviously were not governed by stress distribution but which were primarily influenced by irregularities in the compressibility of the soil. Examination of old maps showed that both buildings were situated on very recent clay, and on silt deposits of an age of less than 100 yr; they were both found to be built on sites upon which the main branch of the Nile flowed only a century ago. These recent deposits apparently had not been compacted and consolidated by repeated drying and flooding, which may account for their irregular nature.

Structures with a theoretically regular distribution of settlements were founded on older clay and silt deposits more than 800 yr old for Buildings III, IV, V, VII, X, and *T*, and deposits more than 2 000 yr for Buildings I and II. They were subjected to repeated drying and flooding during this period, which presumably had a consolidating effect on the degree of compressibility over large areas.[4]

Thus, on a total number of eleven structures discussed in this paper two were an exception to what appears to be a local rule concerning the horizontal uniformity of soil compressibility. The writer wishes to emphasize that, as yet, very little is known concerning the degree of this uniformity for different types of soil deposits in various localities. Furthermore, records from three or four, or even from a dozen, buildings in one locality cannot be considered as sufficient for the purpose of establishing rules for actual designs. Much more extensive information is required; and it can be obtained only by numerous and systematic observations in the respective localities. Until such information becomes available, foundation designs should not be based exclusively on any laws of pressure distribution but should also take into account the most unfavorable conditions that may be created by any assumed possible irregular variations in the compressibility of the soil over the area of the structure.

The settlements described in this paper were observed mostly by means of optical levelings and observation bench-marks systems devised by Professor Terzaghi,[6] several hundred of which were built in, and were found to be most satisfactory in use. Their number generally varied from ten to sixty per building, depending on its area. Bench-marks of the network of the Survey of Egypt served as datum in Cairo. The accuracy of the leveling results reported is about 0.1 in.

Damages.—The damages caused to the buildings by the settlements were found to depend mainly on the radius of curvature of the superstructure, created by its adaptation to the surface of the settlement crater. This confirmed previous investigations of the subject.[7] Only two buildings (Building *T* and VIII) suffered damages. The remainder did not show any cracks, although some settled considerably.

Thus, Building IX settled more than 6 in. with a uniform tilt to one side but without any damage. Buildings I, III, V, VII, and X had differential settlements within their small area which varied from $\frac{1}{2}$ in. to 1 in.; the long Building IV had a differential settlement, between its center and its ends, of about 1.5 in.; the radius of curvature created by these differential settlements was not sufficient to crack the superstructure which deformed without external signs of damage. The maximum settlement in all these cases varied from 1 in. to 3 in.

The radius of curvature and the differential settlement of Building VIII are not known, but they must have been considerably more than those of the preceding cases in proportion to the greater maximum settlement of this building of more than 4 in.

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[6] *Proceedings*, Am. Soc. C. E., September, 1937, Fig. 42(*a*) and Fig. 42(*b*), p. 1360.

[7] *The Structural Engineer*, Lond., March, 1935, p. 148.

Numerous cracks were observed, mainly in partition walls. These cracks were particularly pronounced at the corners of the building, which, in accordance with the theoretical stress distribution, should have a tendency to settle less than the center of the building, severe stresses being created thereby at the corners in the superstructure. The inclination of the cracks supported this conclusion.

The old rule-of-thumb—"strengthen the corners"—should be recalled in this connection and, if correctly interpreted, should refer only to the superstructure. Some engineers understand it as meaning that more piles should be driven or that smaller unit pressures on the soil should be used at the corners. This would have an opposite effect to the one desired, by still further increasing the difference of settlements when structures are founded on horizontally homogeneous soil.

Building *T* was also strongly cracked as a result of its large maximum settlement (10 in.) and of large differential settlements. The separation of its parts of different heights and weights, by means of expansion joints, remained totally ineffective. In this connection it should be noted that many older buildings have been damaged in Cairo when situated partly within the limits of the settlement crater of the adjoining new structures. Harmful effects of the settlement crater were noticeable even when the older structures were separated from the new ones by a free space of 6 to 12 ft.

Future Trends.—The influence of the settlement studies on the new trends of local engineering practice was considerable. In 1934, previous to beginning these studies, the height of buildings in Cairo did not exceed eight or nine stories. Some of them suffered damages, but opinions were strongly divided as to the causes. The observations of the Laboratory provided logical explanations for most of the known facts, disclosed many new ones, and dispelled many erroneous conceptions. In particular, the observations showed how strongly settlements were reduced when the loads of the structure could be transferred to deeper sand layers in a manner not endangering their state of lateral confinement. As a result, the construction of buildings (on long piles driven into the sand) of ten, twelve, and fourteen stories was started in rapid succession, and investigations for an eighteen-story building have been begun.

A few engineers now go to the other extreme and maintain that the loads of all structures should be transferred to sand by means of long piles. This view ignores considerations of economy and, therefore, is just as exaggerated as the previous local antagonism to long piles because of their excess cost.

The observations disclosed the total weight of different types of structures which could be safely supported by various successive soil layers. As a result smaller structures also benefited from these observations, greater assurance and, therefore, more justified economy becoming possible for their designs.

CONCLUSIONS

Systematic settlement observations of full-sized structures are the only reliable means for obtaining accurate data on the actual behavior of these structures and also concerning the soil on which they were erected. In many respects present-day laboratory testing methods and procedures of foundation design are still in need of correction and of development through comparisons with the results of such full-scale observations. Results obtained from observations in one locality are not necessarily valid in other localities. Therefore, such settlement studies should be undertaken in all large cities. This necessity has been recognized generally for some time, but little has been done largely because individual efforts are inadequate as compared to the scope and magnitude of the work to be performed.

The required large scale of such studies necessarily calls for a co-operative effort in their organization. To be scientifically effective these studies should be made by independent organizations not having any business connections with the structures investigated. Special guaranties to the business interests concerned are then necessary, safeguarding them against damages and thereby enlisting their co-operation. The studies in Egypt described in this paper show that such a plan is feasible and can be made successful in practice.

ACKNOWLEDGMENTS

The writer wishes to emphasize the generous support given by the Egyptian Government to the investigations described. He also wishes to express acknowledgments to the persons who have facilitated the development of these studies: To Hussein Sirry Pasha, Minister of Public Works, Egyptian Government, under whose instructions and direction the first scientific settlement investigation in Egypt was made; to Professor Terzaghi who had first proposed systematic settlement studies on full-sized structures and who, through personal contacts, contributed many valuable new ideas and inspiration; to Dr. C. Andreae, former Dean, Faculty of Engineering, Egyptian University, for his support in the organization of the new laboratories; to Dr. W. S. Hanna, Head of the Foundation Soils Research Laboratory, Egyptian University, in close collaboration with whom most of the work described was done[8] and to whose courtesy the writer is indebted for data concerning the continued settlements observed during 1937; to Dr. F. E. Relton, Professor of Mathematics, Faculty of Engineering, Egyptian University, for his help in the treatment of some mathematical problems; to Kamal Khalifa Effendi, former Research Assistant, Foundation Soils Research Laboratory, Egyptian University, for his assistance in making some of the borings, tests, and computations; and to many engineers and architects of Cairo, and of the Provinces, without whose co-operation this work could not have been successfully developed.

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[8] *Bulletin No. 1*, Soil Mechanics and Foundation Research Laboratory, Egyptian Univ., Faculty of Eng., October, 1938.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

ENERGY MASS DIAGRAMS FOR POWER STUDIES

By John W. Hackney,[1] Jun. Am. Soc. C. E.

Synopsis

A method of applying the mass-diagram principle to complex hydro-electric power generation systems is developed and illustrated in this paper. This application involves the computation of potential energy inflow to the system, the segregation of energy lost due to friction, leakage, lack of capacity, evaporation, etc., and the study of the storage, waste, and use of the remainder by the mass-diagram method. That part of the energy which is storable is segregated by using two diagrams concurrently, one of total energy, the other of unstorable energy only, this device being necessary to insure against considering power developed at run-of-river stations to be storable.

General Problem

The mass diagram as a "tool" for engineering analysis is familiar to the profession. In hydro-electric work it has been commonly used to compute the effect of storage on stream flow, and when used in this manner is simply the integral of the stream-flow record, or hydrograph, with respect to time. The outflow in terms of cubic feet per second, as determined from the study of this integral curve, is converted into kilowatts of power output by the use of a power-outflow diagram or a mean power constant. (This mention of the standard mass-diagram method is necessarily abbreviated. A complete discussion is presented elsewhere.[2]) This method of procedure is applicable to a single plant or to a storage plant above one or more run-of-river plants. If other storage plants are added, more complex systems develop. Hydro plants with little or no storage capacity are tied to high-head storage plants and the systems thus formed are tied into other systems on other water-sheds, so that a system may have plants of various storage characteristics, heads, and capacities using the water of several different streams.

Note.—Written comments are invited for immediate publication; to ensure publication, the last discussion should be submitted by **December 15, 1938.**

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[2] "Hydro-Electric Handbook," by William P. Creager and Joel D. Justin, Members, Am. Soc. C. E., p. 99.

It is desirable, if possible, to combine the power characteristics of such a system into a single diagram. This diagram should be adaptable to studies of power output under various load and water conditions, and to studies of various alterations or additions to the system. Such a diagram may be obtained by reducing the various stream-flow mass diagrams of the separate plants by the common denominator, power, to what has been termed an energy mass diagram.[3]

BASIC THEORY

The energy mass diagram measures the cumulative inflow of usable energy to the system with respect to time. When a cubic foot of water has entered a pond or a reservoir, it has a potential energy equivalent to its own weight falling from the height at which it entered the reservoir through the developed head of all plants through which it may subsequently pass. This potential energy may be stored, wasted due to lack of load or capacity, lost in friction, leakage, and evaporation, or converted into useful work. If the mean energy losses are segregated from the initial potential energy, and the remainder is plotted as an energy mass diagram, this diagram may be applied to the study of the storage, waste, and use of the available energy.

An energy mass diagram is the cumulative sum of the energy inflow (corrected for losses) to the hydro-electric system. It has the ordinary properties of a mass diagram. Being the integral of energy inflow, its slope is the rate of inflow and its ordinates are quantities of energy, or storage. In the usual hydro system, it is made up of two kinds of energy—storable and unstorable. Flow from uncontrolled parts of the water-shed represents unstorable energy and must be used as it comes. Flow from areas up stream from reservoirs may be stored to the extent that reservoir space is available. These classes of energy may be segregated by plotting not one but, two, energy mass diagrams. The first is the complete mass diagram, the cumulative sum of all energy inflow to the system. The second, an auxiliary curve, is the cumulative sum of the unstorable energy only.

The application of this basic theory is best explained by an illustrative example. The system chosen for the purpose is a simple one, consisting of one run-of-river plant and one head-water storage plant. It is believed, however, that the discussion of this limited system will reveal the salient features of more complex studies.

Hydro-electric power studies may be divided into three parts, the determination of stream flow available, the determination of load requirements, and the correlation of stream flow to load. The energy mass diagram is designed to be an aid in the study of the last-mentioned phase; therefore, the first two phases are not emphasized in the illustrative problem. This problem has been designed to indicate some of the uses of the energy mass diagram, given the stream flow and the load requirements.

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[3] See, also, the "Mass Curve of Equivalent Flow," described by Don Johnstone, Jun. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 98 (1933), pp. 1294–1295.

ILLUSTRATIVE PROBLEM

Fig. 1 shows the water-shed of "Typical River"[2] in plan and profile. This river has the flow characteristics of a large stream of the South. "Type Creek," one of its tributaries, drains a fairly high plateau region, and, therefore, has quite a different run-off. The average monthly flows, listed in Table 1, were computed from the actual stream-flow records of corresponding streams.

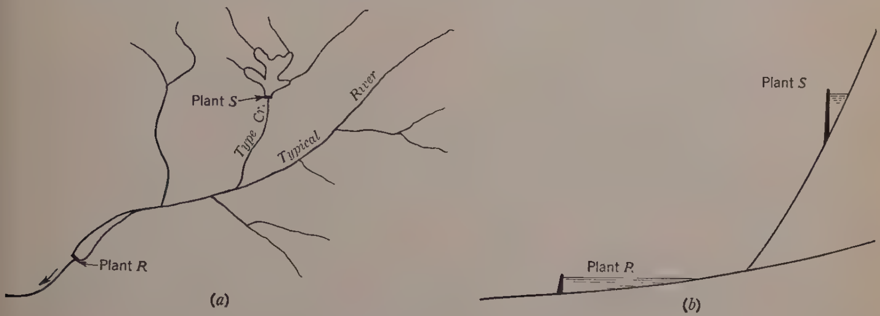

FIG. 1.—"TYPICAL" VALLEY PLANTS

TABLE 1.—AVERAGE MONTHLY FLOWS COMPUTED FROM ACTUAL STREAM-FLOW RECORDS

Month (1913)	NATURAL FLOW, IN CUBIC FEET PER SECOND*			AVAILABLE ENERGY INFLOW, IN THOUSANDS OF KILOWATTS		SUMMATIONS,† IN THOUSANDS OF KILOWATT-MONTHS	
	Main river, Plant *R*	Tributary, Plant *S*	Uncontrolled flow at Plant *R*; Column (2) minus Column (3)	From Plant *S*, acting through Plants *R* and *S*	From Plant *R*	Column (6) minus 240	Column (5) plus Column (6) minus 240
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
......	......	......	......	......	......	2 000.0‡	2 000.0‡
January	40 300	2 760	37 540	138.0	280.0	2 040.0	2 178.0
February	31 900	3 490	28 410	174.5	280.0	2 080.0	2 392.5
March	49 400	6 390	43 010	319.5	280.0	2 120.0	2 752.0
April	26 500	3 450	23 050	172.5	230.5	2 110.5	2 915.0
May	14 600	2 560	12 040	128.0	120.4	1 990.9	2 923.4
June	13 600	1 960	11 640	98.0	116.4	1 867.3	2 897.8
July	5 420	1 380	4 040	69.0	40.4	1 667.7	2 767.2
August	5 000	1 380	3 620	69.0	36.2	1 463.9	2 632.4
September	3 730	1 120	2 610	56.0	26.1	1 250.0	2 474.5
October	3 570	1 180	2 390	59.0	23.9	......	2 317.4
November	3 900	1 140	2 760	57.0	27.6	......	2 162.0
December	5 290	1 460	3 830	73.0	38.3	......	2 033.3

* Corrected for evaporation. † Summations are for the end of the month.
‡ Arbitrary initial value on January 1.

On the main stream, well below the outlet of Type Creek, is a fairly large run-of-river plant, Plant *R*, with adequate pondage, but no storage. Provision has been made here for the installation of additional units when and if they are required. Now, in the course of events, another company acquires a reservoir site on Type Creek and plans a storage project to be known as Plant *S*.

The fundamental data concerning the two plants are as follows:

Description	Plant *R*	Plant *S*
Drainage Area, in Square Miles:		
Controlled	1 000	1 000
Not controlled	9 000	0
Total	10 000	1 000
Gross head, in feet	145	620
Capacity, in thousands of kilowatts	280	190
Storage, in thousands of acre-feet (pondage only)		603

The general problem is to determine the total increase in useful power output that will be produced by operating the new storage plant in conjunction with Plant *R*.

COMPUTATION OF ILLUSTRATIVE ENERGY MASS DIAGRAM

In order to construct energy mass diagrams for the "Typical Valley" plants, it will first be necessary to estimate the losses that should be deducted from the initial quantity of potential energy. Considering the storage plant, *S* (Fig. 1), the following major losses should be taken into account: (*a*) Evaporation; (*b*) loss of head due to draw-down; (*c*) pipe-line losses; (*d*) turbine losses; and (*e*) electrical losses. These items are variable with reservoir elevation, output, and other factors too numerous to mention. It is necessary to assume values for these losses that either are constants or that will vary only with natural phenomena, such as stream flow and time of year, which are not affected by the operation of the system. For Plant *S* the following assumptions have been used:

(1) Evaporation from the reservoir is a fixed quantity for any given month of the year;

(2) Loss of head due to draw-down is a mean quantity equivalent to the head lost when four-tenths of the volume of the reservoir is drawn (in this case, 30 ft);

(3) Pipe-line losses have a mean value of 20 ft;

(4) Turbine efficiency is 90%; and,

(5) Generation efficiency is 95 per cent.

Considering the foregoing losses the power output at Plant *S* may be determined. Let P = electrical power output; e = conversion efficiency; H = mean net head; Q = stream flow (corrected for evaporation); and, k = the power constant of the plant. Then,

$$P = 0.0846\, H\, Q\, e = k\, Q \qquad (1)$$

By Equation (1), $P = 0.0846 \times (620 - 20 - 30)\, Q \times 90\% \times 95\% = 40\, Q$ kw; and $k = 40$ kw per cu ft per sec. Similar assumptions as to losses have been made for the run-of-river plant (*R*, Fig. 1): (1) Evaporation is a fixed quantity for any given month; (2) the mean loss of head due to draw-down is 1 ft; (3) the mean conduit loss is 1 ft; (4) turbine efficiency is 87%; and (5) generation efficiency is 95 per cent.

Therefore, by Equation (1): $P = 0.0846 \times (145 - 1 - 1)\, Q \times 87\% \times 95\% = 10\, Q$ kw; and $k = 10$ kw per cu ft per sec. This factor, k, applies only to normal conditions of stream flow. If the flow is such that the capacity of the turbines is reached, or the head is reduced by high tail-water, k is no longer valid. For these conditions a curve of stream flow *versus* power is developed in three steps:

(*A*) Construct a curve of head *vs* capacity from turbine and generator characteristic curves;

(*B*) Construct a curve of head *vs* total stream flow by using head-water and tail-water rating curves; and,

(*C*) Construct a curve of stream flow *vs* capacity, using the curves developed in Steps (*A*) and (*B*).

In Step (*B*), since capacity limits are being determined the total stream flow is used with the tail-water curve; but this is reduced by the approximate turbine discharge in determining the head-water elevation from the rating curve of the dam.

Using the aforementioned constants, the energy mass diagram is developed using a computation table similar to Table 1, in which Column (5) gives the available energy from the inflow to Plant S acting through Plants R and S; that is, Column (3) times the sum of k-values for the two plants ($40 + 10 = 50$). Column (6) shows the available energy from the uncontrolled inflow to Plant R (Column (4) times $k = 10$). The effects of capacity limitations have been approximated. Column (7) is the summation for the auxiliary diagram; that is, the cumulative summation of Column (6) less the sum of the means of Column (5) and Column (6); and Column (8) is the summation for the main diagram, or the cumulative summation of Column (5) and Column (6), less the sum of the means of these columns.

Columns (7) and (8), Table 1, require some further explanation. It is convenient to plot energy mass diagrams in terms of cumulative excess over the mean, or an even value close to the mean. This produces a mass curve which moves in a generally horizontal direction and makes it possible to enlarge the vertical scale greatly without throwing the curve off the paper. It also produces more definite points of tangency and a more readable curve. For this reason, in Columns (7) and (8) the mean monthly energy inflow for the entire period of record is subtracted from the energy inflow for each particular month before it is added to the cumulative sum. This device usually makes it necessary to assign an arbitrary initial value to the cumulation in order to avoid negative quantities. The auxiliary diagram is plotted to the same mean as the main diagram in order that one slope scale may be used for both. Since the auxiliary curve is normally used only in wet seasons, it may be broken conveniently in the dry season each year to keep it from running off the paper.

COMPUTATION OF OUTPUT OF ILLUSTRATIVE SYSTEM

With the energy mass diagrams computed, the next step in the solution of the problem is to determine, by means of these diagrams, the maximum useful output that would have been produced by the two plants if they had been operated in conjunction for the period of stream-flow record. This requires a thorough-going investigation to determine the most efficient method of correlating stream flow and load. A discussion of the various methods of reservoir control and system operation is beyond the scope of this paper. There appear to be as many "criteria" of operation as there are men interested in their development and use, but one and all may be tested and tried in a manner similar to that used for the trial of Typical River Operation Criterion, as outlined subsequently herein.

It is assumed that the Typical Valley plants have two types of load available: (1) The public utility load of two fairly large towns; and (2) a large electro-chemical industrial load. Continuity of service is the prime requisite of the public utility load. This power must be available at all times, and especially in the dryest years every effort must be made to protect the prime output required for this portion of the load. On the other hand, the electro-chemical industry finds it possible to use power which, although normally continuous, is nevertheless interrupted at long intervals by periods of sub-normal stream flow. Demand for the product during these dry periods will be satisfied from stock piles accumulated during the wet periods.

With such loads available for the system, it is proposed that a two-rate plan of operation be used. The higher, or normal rate, will be carried continuously except during abnormally dry years. In these dry years the prime, or utility, load only will be carried. It will be necessary, then, to develop some form of criterion that will indicate the proper times to drop and resume the secondary load. The drops from the normal rate must be timed so that the prime rates may be guaranteed through any sub-normal flow period; but, on the other hand, both drops and resumptions must be made in such a manner that the greatest possible total value of output may be obtained.

TYPICAL RIVER OPERATION CRITERION

The following criterion, although not the most efficient available, was chosen for its simplicity:

(1) There shall be designated a "normal rate" and a "prime rate" of power production;

(2) Power in excess of the designated "normal rate" shall be produced only from water which it is impossible to store; and

(3) Stored water may be used in generating power at any rate which does not exceed the normal, except when estimates show that, with the available stored water and assumed continuance of the prevailing stream flow conditions, the normal rate cannot be continued: For three months after July 1; for three months after August 1; for three months after September 1; or for one month after the first of any month.

When estimates have shown such a condition to exist, no storable water shall be used to produce power in excess of the prime rate until, at the end of a 30-day period in which the average stream flow has been greater than that necessary to produce the normal power, the reservoirs of the system have been completely filled. Then, only, shall the normal rate of power production be resumed.

Fig. 2(*a*) is the energy mass diagram for the Typical River system plotted from Table 1 for 1913 and from similar computations not included herein for the years 1914–1917. (In an actual problem the entire period of record should

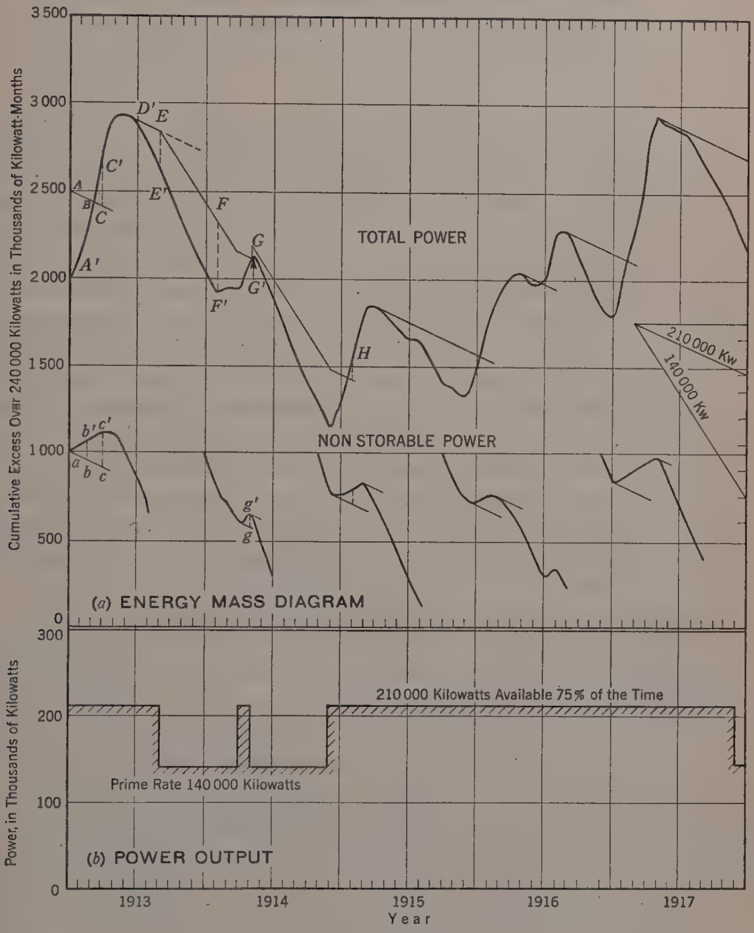

FIG. 2.—STORAGE VALUATION FOR "TYPICAL" RIVER PLANTS

be considered, together with any additional information available concerning minimum flows.) The upper or main curve is the total usable energy inflow to the system, plotted in terms of excess over the mean. The lower, auxiliary, curve in Fig. 2(*a*) is the summation of unstorable energy only. It is plotted in terms of excess over the same mean. To the right of the curves is a slope diagram which is for the purpose of determining the rates of energy inflow and use corresponding to slopes of the mass curves. The vertical scale on the extreme left is used in plotting the curves and measuring storages.

The total amount of energy storage available at Plant *S* is determined by multiplying the storage volume of the reservoir in terms of month-second-feet by the sum of the *k*-values for the stations through which this storage will act. For Plant *S* this will be 603 000 acre-ft, or 10 000 month-sec-ft, times 50 = 500 000 kw-months of storage. With this storage, a determination of the output of the system will be made, applying the operation criterion, with a normal rate of 210 000 kw and a prime rate of 140 000 kw, to the mass diagrams of Fig. 2(*a*).

Assume that on January 1, 1913, the reservoir is at its point of maximum draw-down, and the system is operating at its normal rate. Ordinate *A A′*, Fig. 2(*a*), represents the reservoir depletion (500 000 kw-months), and the slope of Line *A C* represents the normal output rate (210 000 kw). Since the slope is less than that of the mass curve, inflow exceeds outflow and water is being stored at Plant *S*. If all the energy inflow to the system were storable, Point *B* would indicate the time at which the reservoir would be filled. However, if Line *a c*, which has a slope equivalent to the normal output, is drawn on the auxiliary diagram of unstorable energy, it will be found that the unstorable flow at Plant *R* is more than sufficient to provide the requisite 210 000 kw and some of this flow must be wasted. Ordinate *b b′* indicates that at this time the unstorable energy in excess of that used in carrying the normal rate has amounted to 170 000 kw-months since the first of the year; that is, 170 000 kw-months have been wasted or used as dump power, and, therefore, the reservoir is not filled by this amount. The reservoir will be filled only when the rate line, *A C*, has fallen below the mass curve a distance equal to the Ordinate *c c′* between the auxiliary curve and the rate line, *a c*.

From Point *C′* to Point *D′* Fig. 2(*a*), the energy inflow to the system still exceeds the normal outflow and since no more water can be stored a quantity of dump power is produced. The exact amount may be most easily determined from the tabulation of power inflows, making corrections for capacity limits at both plants. From Point *D′* to Point *E* outflow exceeds inflow and the normal rate is being produced by draft from storage, the storage depletion at any particular time being the ordinate between the rate line and the mass curve. At Point *E′* it will be found that, projecting Line *D′ E* and the slope at Point *E′* for three months, the ordinate at that point would exceed the storage available. This means that, if the present stream flow were to continue for three months, the normal output rate would exhaust the available storage. According to the criterion operation, this means that power production should be reduced immediately to the prime rate, represented by the slope, *E F*. Continuing on this new rate, the maximum storage used is *F F′* —about 420 000 kw-months.

The prime rate is used until April, 1914, when the run-of-river power at Plant *R* alone is sufficient to produce the normal rate. The question then arises at *g′* whether to continue at this rate or to drop again to the prime. On the auxiliary curve two tangents are drawn at a slope representing the normal rate. One is at the point where the slope of the auxiliary curve exceeds this rate, and the other where the normal rate exceeds that of the auxiliary curve. At Point G' on the mass curve the rate line has fallen below the curve, but subtracting the energy $(g\,g')$ produced at Plant R in excess of the normal rate by laying off this ordinate above Point G' it may be seen that the reservoir is not full and, therefore, stored water should not, according to the criterion, be used for the production of power in excess of the prime rate. Therefore, the normal rate is not resumed until the first part of December, when the run-of-river power is again sufficient to produce the normal rate. Stored water may again be used in the production of the normal rate after the reservoir is filled, at Point H, Fig. 2(a).

In like manner the remainder of the power graph (Fig. 2(b)) has been developed. To illustrate the methods used, only one determination of power output has been made. For a single actual problem the writer has made thirty-seven such determinations, using various criteria and different normal and prime rates. These results were analyzed graphically in order to select the best operating method. Assuming that this has been done for the Typical River plants, and that the criterion presented herein with a normal rate of 210 000 kw and a prime rate of 140 000 kw has been selected, it is possible to determine the increase in useful power output produced by operating storage Plant S in conjunction with Plant R.

Without the storage plant, but using available pondage at Plant R, the prime output is limited to the average output of the lowest week, which occurred in October, 1913, when the output was 23 000 kw. Duration curve studies indicate that a rate of 54 000 kw was available 75% of the time. Tabulating these data and the data from Fig. 2(b):

Description	Thousands of kilowatts of prime power available 100% of the time	Thousands of kilowatts of power available 75% of the time
With Plant S.........	140	210 − 140 = 70
Without Plant S......	23	54 − 23 = 29
Gain............	117	41

These data represent the power gain produced by building Plant S and operating it in conjunction with Plant R. Attention should be called to the fact that the actual gain is slightly greater, since the power available 75% of the time from Plant R alone is of poor quality, being interrupted at frequent intervals, as compared to the power available from the combined plants.

Conclusion

The illustrative problem has indicated a few of the uses of the energy mass diagram. It is not suggested as a cure-all for power studies. It is another "tool" to be used with analytical methods, the energy draw-down diagram, the hydrograph, the duration curve, and the area-duration curve. Each of these methods is useful in its own field, and the water power engineer should at least be familiar enough with all of them to select the one best suited to his own needs.

The chief advantage of the mass diagram is the great rapidity with which power determinations can be made, once the original computations have been completed. Every method of operation can be given a fair trial, and the engineer is encouraged to originate new methods by the ease with which they can be tested.

For a strict solution, every new plant or change in head would require a new diagram. This objection may be overcome by the use of proportion. The shape of the diagram is assumed to be typical of the water-shed, the addition of plants and changing of heads being represented by changes in scale. Used in this manner, the diagram becomes available for preliminary investigations, the determination of economic development limits, and other less refined calculations.

Limitations which must be borne in mind are the use of: (1) The graphical solution; (2) mean values for losses; and (3) weekly or monthly averages. Limitation (1) may introduce some errors due to paper shrinkage, inaccuracies in slope measurements, etc. However, the graphics may be checked analytically if necessary, using the summation columns for this purpose. Limitation (2) is a necessary feature of the method and may require special investigation of critical periods, such as the weeks of minimum stream flow. With judicious selection of these mean values, this feature will cause little trouble. The errors from Limitation (3) are common to all methods and have been thoroughly discussed elsewhere.[2]

In this presentation of the energy mass diagram no pretense is made of covering all the uses of the method. Ingenious minds will discover other devices and technical improvements which will alter it to fit their particular problems. The energy mass diagram has evolved from the ordinary mass diagram, and it may be hoped that even better methods of power computation will grow from the energy mass diagram method as herein set forth.

ACKNOWLEDGMENTS

The writer wishes to acknowledge his indebtedness to James P. Growdon, Robert F. Ewald, and Harold A. Thomas, Members Am. Soc. C. E., and also to the great number of other engineers whose ideas and methods have been consciously or unconsciously incorporated in this paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

BEAM CONSTANTS FOR CONTINUOUS TRUSSES AND BEAMS

By George L. Epps,[1] Jun. Am. Soc. C. E.

Synopsis

The material presented in this paper deals with the determination of the elastic constants used in various methods of analyzing continuous beams and trusses. By rotating the ends of the beam in turn and computing the elastic curve, the constants are determined for a span. This method simplifies greatly the process of determining stresses in continuous trusses and girders.

Introduction

The method of determining beam constants by elastic weights, described herein, was first developed for use in the analysis of continuous beams of variable moment of inertia and, later, was extended to include continuous trusses. Since the application of this method to continuous trusses is more involved, it will be explained in detail. The simpler application to continuous beams will be indicated afterward.

The usual method of analyzing continuous trusses is to assume various reactions as redundants and compute the elastic curves with loads in their places. In Fig. 1(*a*), Curve *A E F D* is the deflection curve for a unit load at Point *B*; Curve *A G F D* is the deflection curve for a load of sufficient magnitude at Point *C* to produce a deflection at that point equal to *C F*; and the shaded area between these two curves represents the ordinates of the required influence line to some scale. These deflections are several times larger than

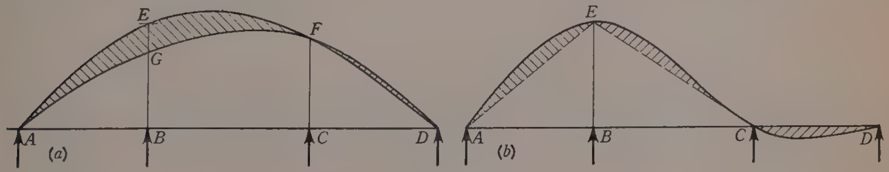

Fig. 1

Note.—Written comments are invited for immediate publication; to ensure publication, the last discussion should be submitted by December 15, 1938.

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the influence ordinates that are obtained from them and, therefore, must be determined quite exactly if the ordinates are to be accurate.

A very clear illustration of the necessity for great exactness is given in an example of a 3-span continuous girder analyzed by Mr. V. L. Pierce[2] in 1935. In Fig. 2 the deflections at the pier, R_2, for the loads shown are 4.2666 and -3.2666, and the influence ordinate is the difference, or 1.0000. The larger of these deflections is more than four times the required ordinate and must be accurate to 0.025% if the ordinate is to check within 0.1 per cent. In a 5-span continuous girder, analyzed in the Design Department of the Kansas Highway Commission, the larger deflections were thirty-three times the ordinate, thus requiring very precise calculations to obtain the correct value of the influence ordinate. Thus, it will be seen that the deflection (and hence, the necessary precision of calculations) is increased in proportion to the fourth power of the number of spans.

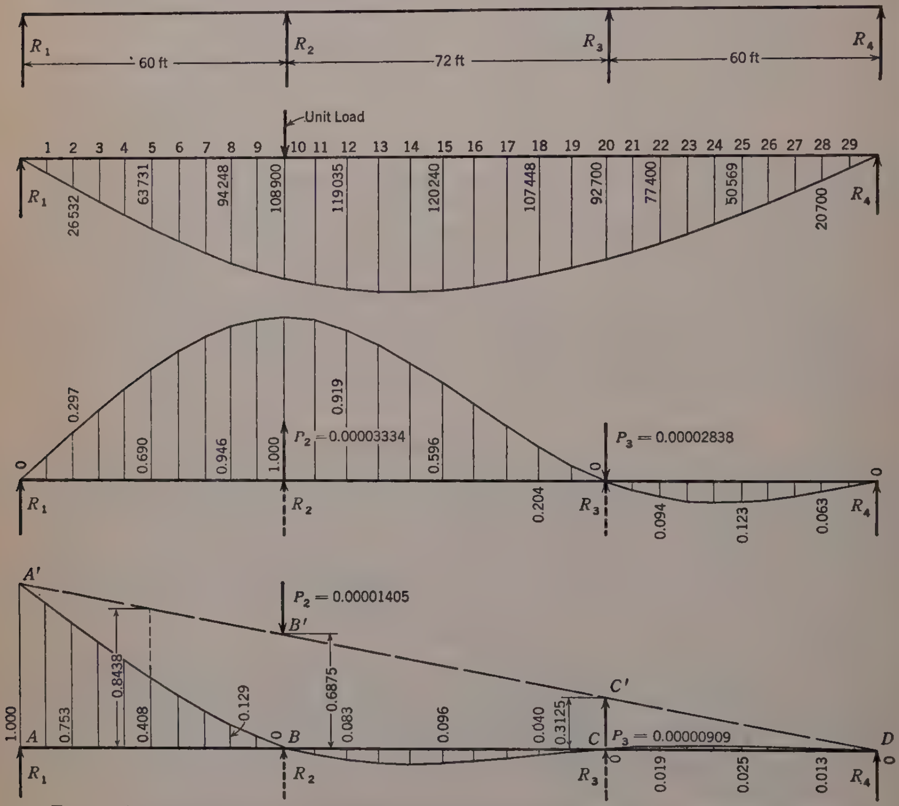

FIG. 2.—DETERMINATION OF INFLUENCE LINES FOR THE REACTIONS OF A CONTINUOUS BEAM

The method explained herein makes possible the application of moment distribution, slope deflection, conjugate points, and similar methods, to the analysis of continuous trusses. Using these methods the moments are determined at each pier. From these pier moments the changes in reactions due to continuity are found. These changes in reactions are indicated by the shaded areas of Fig. 1(*b*). The dashed line of this diagram is the influence line for R_B if there is no continuity at the piers, and the solid line or curve becomes the influence line for R_B of a 3-span continuous beam. The part of the influence ordinate between the dashed and solid lines is not more than one-third the total ordinate, and, consequently, an error of 0.3% in this part will reduce to 0.1%, or less, of the total. Inasmuch as slide-rule computations can easily be made to an accuracy of 0.3%, this method readily lends itself to the use of the slide-rule. The first time the method was applied by the writer to a continuous truss (a 5-span bridge) the influence lines for moments, shears, and reactions at each pier were obtained in one week less than the customary office time. The reaction lines thus found, checked within 0.1% with those computed by other designers who were using the method of redundant reactions. The method of redundant reactions required two weeks to find the influence lines for reactions only.

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[2] "Simplified Solution of Influence Lines for the Reactions of Continuous Beams," by V. L. Pierce, *Civil Engineering*, December, 1935, p. 798.

In the analysis of a continuous truss or girder by slope-deflection or moment distribution methods, the stiffness, carry-over factor, and influence line for fixed-end moments are required for each end of each span. All these quantities may be obtained by the use of elastic weights. According to Müller-Breslau's principle, if a unit rotation is applied to the end of a beam, the opposite end being freely supported, the deflections produced will be the ordinates of the influence line for fixed-end moment. The moment required to produce this rotation is defined by Hardy Cross,[3] M. Am. Soc. C. E., as the stiffness of the beam. The rotation at the far end of the beam is the carry-over factor for moments from the far end to the near end of the beam. The foregoing constants may be used directly in the slope-deflection equations and, with proper adjustment, for the case in which both ends are fixed, in the moment distribution method. All calculations are made using elastic weights as described by D. B. Steinman,[4] M. Am. Soc. C. E., or by Hale Sutherland and Harry L. Bowman, Members, Am. Soc. C. E.[5]

THEORY

Consider the second span of a 5-span continuous truss bridge with members that are symmetrical as to length, but not as to cross-section, so that the elastic constants and fixed-end moments are different for the two ends. Two sets of computations are required. However, those for only one end will be shown herein.

Fig. 3 shows the truss span diagrammatically, the lengths of the members, their moment centers, and the external forces used in the calculations. These forces are assumed and may have any values consistent with the theory of statics. As will be seen, the left end is subjected to a moment and the right

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[3] "Continuous Frames of Reinforced Concrete," by Hardy Cross and Newlin D. Morgan, pp. 83 and 99, John Wiley & Sons, Inc., 1932.

[4] "Truss Deflections Accurately Determined by Elastic Weights," by D. B. Steinman, *Engineering Record*, May 13 and 20, 1916.

[5] "Structural Theory and Design," by Hale Sutherland and Harry L. Bowman, John Wiley & Sons, Inc., 1930, p. 152.

TABLE 1.—Stresses and Characteristics of the Truss Shown in Fig. 3.

Bar	Stress, s, in pounds	Area, A, in square inches	Length, L, in inches	Values of $\frac{s\,L}{A}$	Radii, r, in inches	Elastic Weights, $\frac{s\,L}{A\,r}$, at the Following Panel Points:												
						8	9	10	11	12	13	14	15	16	17	18	19	20
$U_8\,U_9$	−600	21.16	150	−4 260	180	−23.62	....	....	....	....	....	....	....	....	....	....	....	....
$L_8\,U_9$	−94	19.11	234	−1 150	115 96	+10.00	−12.00	....	....	....	....	....	....	....	....	....	....	....
$L_8\,L_9$	+673	19.11	153	+5 390	147	....	−36.62	....	....	....	....	....	....	....	....	....	....	....
$U_9\,L_9$	+66	11.77	150	+840	1 500	....	−0.56	....	....	....	....	....	....	....	....	....	....	....
$L_9\,L_{10}$	+663	19.11	151	+5 230	149	....	−34.98	....	....	....	....	....	....	....	....	....	....	....
$U_9\,L_{10}$	+9	15.59	202	+120	111 100	....	+1.08	−1.20	....	....	....	....	....	....	....	....	....	....
$U_9\,U_{11}$	−666	19.11	300	−10 440	135	....	....	−77.50	....	....	....	....	....	....	....	....	....	....
$L_{10}\,U_{11}$	−11	21.16	202	−108	100 89	....	....	+1.08	−1.21	....	....	....	....	....	....	....	....	....
$L_{10}\,L_{11}$	+679	19.11	151	+5 350	119	....	....	....	−44.80	....	....	....	....	....	....	....	....	....
$U_{11}\,L_{11}$	+67	11.77	120	+684	1 200	....	....	....	−0.57	....	....	....	....	....	....	....	....	....
$L_{11}\,L_{12}$	+675	19.11	150	+5 290	120	....	....	....	−44.10	....	....	....	....	....	....	....	....	....
$U_{11}\,L_{12}$	−96	11.77	192	−1 570	94	....	....	....	−16.75	+16.75	....	....	....	....	....	....	....	....
$U_{11}\,U_{13}$	−600	24.73	300	−7 290	120	....	....	....	....	−60.70	....	....	....	....	....	....	....	....
$L_{12}\,U_{13}$	+96	13.24	192	+1 390	94	....	....	....	....	−14.86	+14.86	....	....	....	....	....	....	....
$L_{12}\,L_{14}$	+525	25.36	300	+6 210	120	....	....	....	....	....	−51.80	....	....	....	....	....	....	....
$U_{13}\,L_{14}$	−96	11.77	192	−1 570	94	....	....	....	....	....	−16.75	+16.75	....	....	....	....	....	....
$U_{13}\,U_{15}$	−450	29.11	300	−4 630	120	....	....	....	....	....	....	−38.60	....	....	....	....	....	....
$L_{14}\,U_{15}$	+96	11.77	192	+1 570	94	....	....	....	....	....	....	−16.75	+16.75	....	....	....	....	....
$L_{14}\,L_{16}$	+375	24.73	300	+4 560	120	....	....	....	....	....	....	....	−38.00	....	....	....	....	....
$U_{15}\,L_{16}$	−96	15.59	192	−1 190	94	....	....	....	....	....	....	....	−12.63	+12.63	....	....	....	....
$U_{15}\,U_{17}$	−300	19.11	300	−4 710	120	....	....	....	....	....	....	....	....	−39.20	....	....	....	....
$L_{16}\,U_{17}$	+96	13.24	192	+1 390	94	....	....	....	....	....	....	....	....	−14.86	+14.86	....	....	....
$L_{16}\,L_{17}$	+225	19.11	150	+1 760	120	....	....	....	....	....	....	....	....	....	−14.72	....	....	....
$U_{17}\,L_{17}$	+22	11.77	120	+228	1 200	....	....	....	....	....	....	....	....	....	−0.19	....	....	....
$U_{17}\,L_{18}$	+226	19.11	151	+1 790	119	....	....	....	....	....	....	....	....	....	−14.92	....	....	....
$U_{17}\,L_{18}$	−123	24.73	202	−1 010	89 100	....	....	....	....	....	....	....	....	....	−11.31	+10.03	....	....
$U_{17}\,U_{19}$	−133	19.11	300	−2 090	135	....	....	....	....	....	....	....	....	....	....	−15.48	....	....
$L_{18}\,U_{19}$	+99	15.59	202	+1 280	100 111	....	....	....	....	....	....	....	....	....	....	−12.80	+11.51	....
$L_{18}\,L_{19}$	+60	27.86	151	+324	149	....	....	....	....	....	....	....	....	....	....	....	−2.17	....
$U_{19}\,L_{19}$	+6	11.77	150	+72	1 500	....	....	....	....	....	....	....	....	....	....	....	−0.05	....
$L_{19}\,L_{20}$	+61	27.86	153	+336	147	....	....	....	....	....	....	....	....	....	....	....	−2.28	....
$U_{19}\,L_{20}$	−94	19.11	234	−1 150	96 115	....	....	....	....	....	....	....	....	....	....	....	−12.00	+10.00
Totals	....	....	....	......	....	−13.62	−83.08	−77.62	−107.43	−58.81	−53.69	−38.60	−33.88	−41.43	−26.28	−18.25	−4.99	+10.00

end is free to rotate. The stresses are computed by statics and are presented, with the areas, lengths, and radii for the various members, in Table 1. From these values the elastic weights were computed as indicated. The sign convention used assumes that strains which produce a lowering of a joint have negative elastic weights, and *vice versa.*

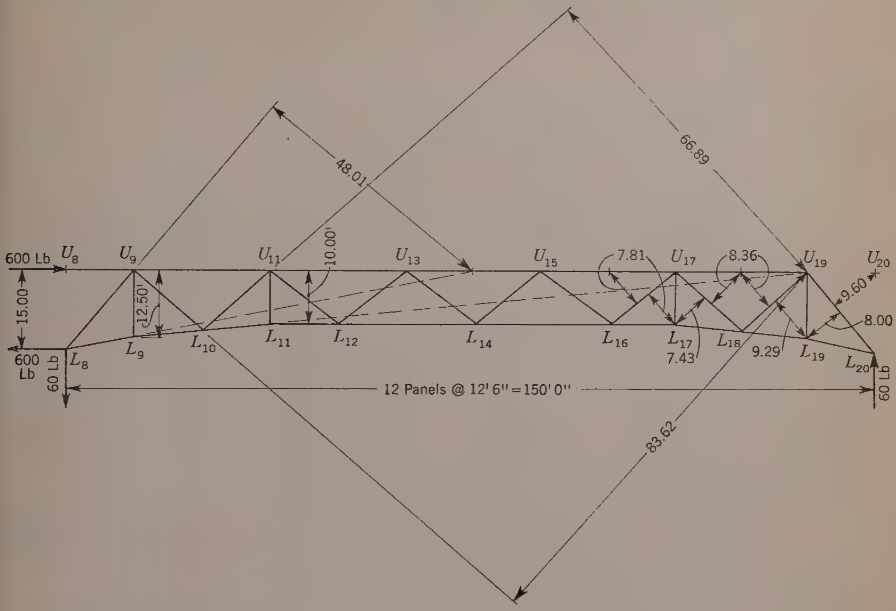

FIG. 3

These elastic weights are applied to a simple or conjugate beam as shown in Fig. 4. In Table 2, the first operation is the determination of the left reaction of the elastic weights by taking moments at the right end of the span. Beginning with the left reaction the shears are computed between load points. These shears are the slopes at successive points of the elastic curve of the truss under this loading.

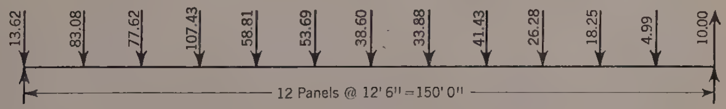

FIG. 4

As indicated previously the end rotation or slope must be unity to obtain the beam constants. Consequently, the first column of elastic shears (Column (5), Table 2) is divided by the end slope of 362.84 to obtain the second column of elastic shears (Column (6), Table 2). From these, the moments (Column (7), Table 2) in the conjugate beam are found by summation. These moments must be multiplied by the panel length to obtain the correct value of the fixed-end moments (Column (8), Table 2). The carry-over is the last shear in Table

2 (Column (6)), or the slope of the far end of the beam, 0.509. The stiffness is $\frac{600 \times 15}{362.84} = 24.79$.

TABLE 2.—Determination of Fixed-End Moments at Point 8.

Panel Point: (1)	Elastic weight (2)	Arm No. (3)	Moment (4)	Elastic Shear (5)	Elastic Shear (6)	$\frac{H_s}{12.5}$ (7)	H_s (8)
				362.84	1.000		
8	−13.62	12	163.44			0	0
				349.22	0.964		
9	−83.08	11	913.89			0.964	12.06
				266.14	0.735		
10	−77.62	10	776.20			1.699	21.22
				188.52	0.520		
11	−107.43	9	966.87			2.219	27.75
				81.09	0.224		
12	−58.81	8	470.48			2.443	30.59
				22.28	0.061		
13	−53.69	7	375.83			2.504	31.35
				−31.41	−0.087		
14	−38.60	6	231.60			2.417	30.20
				−70.01	−0.193		
15	−33.88	5	169.40			2.224	27.81
				−103.89	−0.286		
16	−41.43	4	165.72			1.938	24.22
				−145.32	−0.401		
17	−26.28	3	78.84			1.537	19.21
				−171.60	−0.474		
18	−18.25	2	36.50			1.063	13.30
				−189.85	−0.524		
19	−4.99	1	4.99			0.539	6.75
				−194.84	−0.538		
20	+10.00	0	0			0	0
				−184.84	−0.509		
Total	−547.68	7.94	4 354.75	....	....	....	....

The foregoing values may be used directly in the slope deflection equations when they are written in the following general form:

$$M_{AB} = \frac{1}{1 - r_{AB}\, r_{BA}} \{E\, K_A\, [\theta_A + r_{AB}\, \theta_B - (r_{AB} + 1)\, R] + H_{AB} - r_{BA}\, H_{BA}\} \qquad (1a)$$

and,

$$M_{BA} = \frac{1}{1 - r_{AB}\, r_{BA}} \{E\, K_B\, [\theta_B + r_{BA}\, \theta_A - (r_{BA} + 1)\, R] - H_{BA} + r_{AB}\, H_{AB}\} \qquad (1b)$$

(The general formulas shown for the slope-deflection equations were developed from equations presented by Earle Russell, in "Analysis of Continuous Frames." His equations apply to constants for beams fixed at both ends; whereas, in this method, the constants are developed for one end fixed and the other free. Also, the letters designating the various terms have been changed to agree with those used in *Bulletin 108*, Engineering Experiment Station, University of Illinois.)

In using moment distribution,[6] the constants for stiffness and fixed-end moments must be calculated with both ends fixed for the intermediate spans.

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[6] "Analysis of Continuous Frames by Distributing Fixed-End Moments" by Hardy Cross, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

The stiffness obtained previously must be divided by $1 - r_{AB}\, r_{BA}$. The fixed-end moments, C_A and C_B, are determined by combining the values for H_A and H_B according to the following equations:

$$C_A = \frac{H_A - r_{BA}\, H_B}{1 - r_{AB}\, r_{BA}} \qquad (2a)$$

and,

$$C_B = \frac{H_B - r_{AB}\, H_A}{1 - r_{AB}\, r_{BA}} \qquad (2b)$$

For those who prefer to use graphic methods, it might be shown that all the trusses for Table 1 and the computations of Table 2 may be done graphically by methods found in any text on graphic statics. It should be noted that the elastic loads must be divided by the end reaction to obtain the slope of unity at the rotated end. In using the method of conjugate points,[7] the values of u, v, p, q, s, and t are necessary: $u\, l$ is the distance from the left end of the span to the center of gravity of the elastic loads with a unit rotation at the left end; $v\, l$ is a similar distance for the right end; $\frac{p\, l}{1}$ is the sum of the elastic weights for the left end, rotated (these are the elastic weights of Table 2 divided by the end reaction, or 362.84); and $\frac{q\, l}{1}$ is a similar quantity for the right end. The values of s and t are found by the construction shown in Fig. 5.

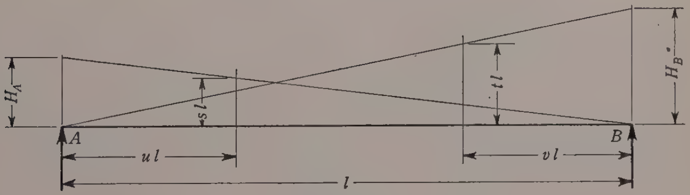

FIG. 5

In some cases it is desired to know the effect of the web system on the stresses in the truss. This can be done easily by omitting the elastic weights for the web members and carrying through the computations in Table 2.

The application of this method to beams of variable moment of inertia differs from the foregoing procedure only in the method of obtaining the elastic weights. The beam is divided into a number of sections, which need not be equal in length (although, if they are, the computations are simplified). Twelve equal sections will produce sufficiently accurate results. A moment is applied to one end of the span with the other end unrestrained and the value of $\frac{M\, dx}{I}$ is computed for each section. These values of $\frac{M\, dx}{I}$ are applied as elastic weights to the conjugate beam, as in Fig. 4. The method proceeds from this point in the same manner as explained previously.

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[7] Discussion of Walter Ruppel, Assoc. M. Am. Soc. C. E., on "Moments in Restrained and Continuous Beams by the Method of Conjugate Points," by L. H. Nishkian and D. B. Steinman, Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 152.

CONCLUSIONS

Multiple-span, continuous truss or girder bridges may be analyzed by the method herein presented in much less time and with less work than is required by the method of redundant reactions. This method makes possible the use of slide-rule computations for continuous bridges, whereas the method of redundant reactions necessitates the use of a calculating machine.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

SOLUTION OF EQUATIONS IN STRUCTURAL ANALYSIS BY CONVERGING INCREMENTS

BY GEORGE H. DELL,[1] ASSOC. M. AM. SOC. C. E.

SYNOPSIS

A method of solving equations in structural analysis by converging increments (which is related to the process of successive substitutions) is demonstrated in this paper. The method is also useful as a guide in developing alternate solutions, of the moment-distribution type, in cases where the proper mode of performing the distributions may not be otherwise readily apparent.

INTRODUCTION

The past decade has witnessed the widespread adoption of the masterly method of moment-distribution,[2] introduced to the Society by Hardy Cross, M. Am. Soc. C. E., in 1930, which has resulted in dispelling to an unprecedented extent the practical difficulties formerly associated with the subject of statically indeterminate structures. At the same time, the new "movement" has produced a stimulation of interest in the older, algebraic methods of analysis, such as three-moment equations, area moments, slope deflection, etc.

This interest appears to be justified. Many individuals have a natural preference for algebraic methods. Besides, the fact that algebraic procedure has played an important rôle in indeterminate analysis in the past would seem to provide grounds for supposing that these methods will continue to exercise a strong appeal to writers and students for a long time to come. However, algebraic analysis has been somewhat handicapped by a scarcity of practical methods of solving the simultaneous equations. Precise methods generally require too much time; hence, the process of successive substitutions, or iteration, has been used to a considerable extent. The latter method is open to the criticisms that it is lacking in form and is applicable to only the most rapidly converging types of equations.

Herein, the writer will attempt to show that the method of converging increments provides an easy, rapid, orderly, and accurate means of solving

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NOTE.—Written comments are invited for immediate publication; to ensure publication, the last discussion should be submitted December 15, 1938.

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[2] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

the equations encountered in a large variety of structural problems; and, furthermore, that it can always be used as a guide to the development of alternate solutions of the distribution type, thereby extending the advantages of this form of solution to new problems in which the distribution ratios cannot be readily determined by inspection. (Applications of the method of converging increments to problems in least squares were described by the writer in 1935.)[3]

EXPLANATION AND APPLICATION OF THE METHOD

The method of converging increments, which is related to the process of iteration, is applicable to systems of simultaneous equations in which each of the several unknowns is strongly represented by a particular equation. As most persons who have worked with the iteration process have doubtless observed, after the completion of a few cycles of substitutions, such systems generally begin to converge with approximately equal rates in the separate equations. When this stage is reached, in the method of converging increments, the solution is brought to a conclusion. If the rate of convergence is rapid, the computations already made are added and the values of the unknowns immediately ascertained. If the rate of convergence is slow, the values of the remaining cycles of figures are computed by series summation, thus making possible a considerable saving in time, while preserving a relatively high degree of precision in the results.

A general derivation of the method as applied to diagonally symmetrical systems has been given elsewhere.[3] The derivation is equally valid for nonsymmetrical systems.

The method is ideally adapted to the use of the slide-rule as each successive line in the solution is computed with a single setting of the slide, and the precision of the method as a whole is consistent with that of the slide-rule.

In order to illustrate the application of the method to problems in structural analysis, three examples are given. For Examples 1 and 2, a continuous beam is solved by means of three-moment equations and also by slope-deflection equations. These examples are used to represent the case of rapidly converging equations. Example 3, which deals with a crane-girder bent, is used to represent the case of slowly converging equations.

Example 1.—The equation of three moments for a continuous beam on unyielding supports and with modulus of elasticity constant throughout, may be written in the form:

$$M_0 K_2 + 2 M_1 (K_1 + K_2) + M_2 K_1 = -\frac{6 \bar{x}_1 A_1 K_2}{L_1^2} - \frac{6 \bar{x}_2 A_2 K_1}{L_2^2} \ldots (1)$$

in which (as shown in Fig. 1): M_0, M_1, and M_2 = the bending moments at the left, center, and right supports, respectively; K_1 and K_2 = the relative $\frac{I}{L}$-values of the left and right spans, respectively; I = moment of inertia;

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[3] "The Adjustment of a Level Net," by George H. Dell, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 834.

L_1 and L_2 = the lengths of the left and right spans, respectively; $\bar{x}_1$ and $\bar{x}_2$ = the distances from the outer supports to the centroids of the two-moment diagrams, the spans being considered as simple beams; and, A_1 and A_2 = the areas of the simple-beam moment diagrams on the left and right spans, respectively. Equation (1) may also be written:

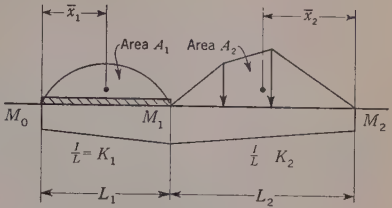

FIG. 1

$$M_0 K_2 + 2 M_1 (K_1 + K_2) + M_2 K_1 = K_2 (C_{01} + 2 C_{10}) + K_1 (2 C_{12} + C_{21})$$

in which C_{01}, C_{10}, etc., the fixed-end moments, are positive for tension in lower fibers.

For the continuous beam shown in Fig. 2, $\frac{\bar{x}_1}{L_1} = \frac{\bar{x}_2}{L_2} = \frac{1}{2}$, and Equation (1) may be written:

$$M_0 K_2 + 2 M_1 (K_1 + K_2) + M_2 K_1 = -\frac{3 A_1 K_2}{L_1} - \frac{3 A_2 K_1}{L_2} \ldots\ldots (2)$$

The relative values of K for the various spans of the continuous beam are shown in parentheses in Fig. 2. Upon applying Equation (2) to successive pairs of adjacent spans in this continuous beam, the following system of equations is obtained:

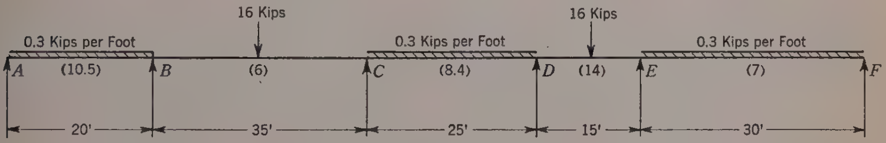

FIG. 2

$$\begin{aligned}
33\, M_B + 10.5\, M_C \qquad\qquad\qquad\qquad &= -\,2\,385 \ldots\ldots (B)\\
8.4\, M_B + 28.8\, M_C + \quad 6\, M_D \qquad\qquad &= -\,2\,045 \ldots\ldots (C)\\
14\, M_C + 44.8\, M_D + \ 8.4\, M_E &= -\,1\,412 \ldots\ldots (D)\\
7\, M_D + \quad 42\, M_E &= -\,1\,575 \ldots\ldots (E)
\end{aligned}$$

Preliminary to solving, Equations (B) to (E) are written in columns, as shown in Table 1(a). In the following description, Lines Nos. 1, 2, 3, and 4, Table 1, will be referred to as the "first line of the equations," the "second line of the equations," etc. It will be noted that the strong coefficient in each of the equations is underlined.

The arrangement of the solution is shown by the first set of calculations (see Table 2(a)), which (for a system of four equations) consists of the first four lines of the solution. In Line No. 1, Table 2(a), the underlined quantity, $-\,\underline{2\,385}$, is the value of the right-hand term of Equation (B). This quantity is extended horizontally in proportion to the coefficients in the first line of the equations, namely, $\underline{33}$, 8.4, 0, and 0. In Line No. 2, the underlined quantity is first computed by subtracting – 607 (in Line No. 1) from – 2 045 (the right-hand term of Equation (*C*)), giving $-\underline{1\,438}$. This value is then extended horizontally in proportion to the coefficients of the second line of the equations, namely, 10.5, $\underline{28.8}$, 14, and 0. A similar procedure is followed in obtaining Lines Nos. 3 and 4.

In ensuing sets, the same order of operation is observed as in the first set. The underlined quantity in a given column is equal to the sum of the extensions entered in that column in the preceding lines of the solution below the last underscored extension, multiplied by − 1. The underlined quantities are always extended horizontally in proportion to the coefficients in the corresponding line of the equations. In this manner, the second and third sets of calculations are found to be as given in Tables 2(*b*) and 2(*c*).

Ordinarily, the foregoing process should be continued until the underlined quantities in the final set are found to be in a practically constant ratio with respect to the corresponding quantities of the preceding set (or until the extended items become negligible). Inspection of the computations in Table 2 shows that up to the third set such a condition has not been attained. Accordingly, a fourth set of calculations is made, as shown in Table 2(*d*).

It is now evident that in the second, third, and fourth columns of the calculations, the terms, $-\underline{8}$, $\underline{5}$, and $-\underline{1}$ are each about one-fifth of the corresponding quantities in the third set, namely, $-\underline{45}$, $\underline{32}$, and $-\underline{5}$, respectively. In the first column, this ratio does not hold; on the contrary, an apparent divergency exists. However, it is seen that if a fifth set of calculations were begun, the underlined quantity in the first column would be $+\underline{3}$, which, when compared to $+\underline{16}$, would give these values approximately the same ratio, or rate of convergence, as has been attained by the remaining columns. Consequently, the solution is terminated at the end of Set 4. The sums of the underlined quantities in the separate columns in Sets 1 to 4, inclusive, are:

Equation	Sum of underlined quantities in Table 2	Equation	Sum of underlined quantities in Table 2
(*B*)...........	− 1 832	(*D*)...........	− 366
(*C*)...........	− 1 528	(*E*)...........	− 1 518

The values of the unknowns are then found by dividing the foregoing sums by the underlined coefficients in the respective equations: $M_B = \dfrac{-1\,832}{33} = -55.5$; $M_C = \dfrac{-1\,528}{28.8} = -53.1$; $M_D = \dfrac{-366}{44.8} = -8.16$; and, $M_E = \dfrac{-1\,518}{42} = -36.1$.

Example 2.—In the slope-deflection method the expression for the bending moment at Point *A* in Member *A B*, in terms of the deformations, is given by Equation (3):[4]

$$M_{AB} = 2\,E\,K\,(2\,\theta_A + \theta_B - 3\,R) + C \ldots\ldots\ldots\ldots\ldots (3)$$

in which E = the modulus of elasticity; K = the value of the ratio $\frac{I}{L}$ (moment

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[4] "Analysis of Statically Indeterminate Structures by the Slope Deflection Method," by W. M. Wilson, F. E. Richart, and Camillo Weiss, Members, Am. Soc. C. E., *Bulletin No. 108*, Section 7, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

TABLE 1.—Arrangement of Equations in Examples 1 and 2

Line No.	(a) Example 1				(b) Example 2			
	(B)	(C)	(D)	(E)	(B)	(C)	(D)	(E)
1	$\underline{33}$	8.4	....	....	$\underline{27.75}$	6	....	....
2	10.5	$\underline{28.8}$	14	....	6	$\underline{28.8}$	8.4	....
3	....	6	$\underline{44.8}$	7	....	8.4	$\underline{44.8}$	14
4	....	....	8.4	$\underline{42}$	....	....	14	$\underline{38.5}$
Total	−2 385	−2 045	−1 412	−1 575	55.00	−54.38	14.38	3.75

TABLE 2.—Solution of Equations (B) to (E), Example 1

Line No.	(B)	(C)	(D)	(E)
	(a) First Set			
1	$-\underline{2\,385}$	−607	....	....
2	−524	$-\underline{1\,438}$	−698	....
3	....	−96	$-\underline{714}$	−112
4	....	....	−293	$-\underline{1\,463}$
	(b) Second Set			
1	$\underline{524}$	133	....	....
1	−13	$-\underline{37}$	−18	....
3	....	42	$\underline{311}$	49
4	....	....	−10	$-\underline{49}$
	(c) Third Set			
1	$\underline{13}$	3	....	....
2	−16	$-\underline{45}$	−22	....
3	....	4	$\underline{32}$	5
4	....	....	−1	$-\underline{5}$
	(d) Fourth Set			
1	$\underline{16}$	4	....	....
2	−3	$-\underline{8}$	−4	....
3	....	1	$\underline{5}$	1
4	....	....	0	$-\underline{1}$

TABLE 3.—Solution of Equations (6), Example 2

Line No.	(B)	(C)	(D)	(E)
	(a) First Set			
1	$\underline{55.00}$	11.90	....	....
2	−13.80	$-\underline{66.28}$	−19.33	....
3	....	6.31	$\underline{33.71}$	10.52
4	....	....	−2.46	$-\underline{6.77}$
	(b) Second Set			
1	$\underline{13.80}$	2.99	....	....
1	−1.94	$-\underline{9.30}$	−2.71	....
3	....	0.97	$\underline{5.17}$	1.61
4	....	....	−0.58	$-\underline{1.61}$
	(c) Third Set			
1	$\underline{1.94}$	0.42	....	....
2	−0.29	$-\underline{1.39}$	−0.41	....
3	....	0.19	$\underline{0.99}$	0.31
4	....	....	−0.11	$-\underline{0.31}$
	Totals of the Underlined Quantities:			
	70.74	−76.97	39.87	−8.69
	Divisors (see Table 1(b)):			
	27.75	28.8	44.8	38.5
	Values of the Unknowns:			
	2.55	−2.67	0.891	−0.226

of inertia divided by the length) for the member; θ_A, θ_B = the changes in slope of the elastic curve at Point A and at Point B, respectively, from its initial position; R = the ratio of deflection to length of member; and, C = the fixed-end moment at Point A due to external transverse forces acting on the member.

A somewhat more convenient form for Equation (3) is obtained by means of the substitutions: $A = 2\,E\,\theta_A$; $B = 2\,E\,\theta_B$; and, $\lambda = 6\,E\,R$. Equation (3) may then be written as follows:

$$M_{AB} = K\,(2\,A + B - \lambda) + C \qquad (4)$$

For a continuous beam on unyielding supports the quantity, λ, is equal to zero in each span. In accordance with Equation (4), the bending moments at the supports of the continuous beam in Fig. 2 may then be expressed as follows:

$$M_{AB} = 10.5\,(2\,A + B) - 10 \qquad (5a)$$

$$M_{BA} = 10.5\,(A + 2\,B) + 10 \qquad (5b)$$

$$M_{BC} = 6\,(2\,B + C) - 70 \qquad (5c)$$

$$M_{CB} = 6\,(B + 2\,C) + 70 \qquad (5d)$$

$$M_{CD} = 8.4\,(2\,C + D) - 15.62 \qquad (5e)$$

$$M_{DC} = 8.4\,(C + 2\,D) + 15.62 \qquad (5f)$$

$$M_{DE} = 14\,(2\,D + E) - 30 \qquad (5g)$$

$$M_{ED} = 14\,(D + 2\,E) + 30 \qquad (5h)$$

$$M_{EF} = 7\,(2\,E + F) - 22.5 \qquad (5i)$$

and,

$$M_{FE} = 7\,(E + 2\,F) + 22.5 \qquad (5j)$$

Upon putting $\Sigma M = 0$ at each of the joints, the following equations of static equilibrium (which may be written by inspection) are obtained:

$$21\,A + 10.5\,B = 10.00 \qquad (6a)$$

$$10.5\,A + 33\,B + 6\,C = 60.00 \qquad (6b)$$

$$6\,B + 28.8\,C + 8.4\,D = -54.38 \qquad (6c)$$

$$8.4\,C + 44.8\,D + 14\,E = 14.38 \qquad (6d)$$

$$14\,D + 42\,E + 7\,F = -7.50 \qquad (6e)$$

$$7\,E + 14\,F = -22.50 \qquad (6f)$$

By combining Equations (6*a*) and (6*b*), (6*e*) and (6*f*), the following system of equations in four unknowns results:

$$27.75\,B + 6\,C = 55.00 \qquad (6b')$$

$$6\,B + 28.8\,C + 8.4\,D = -54.38 \qquad (6c)$$

$$8.4\,C + 44.8\,D + 14\,E = 14.38 \qquad (6d)$$

$$14\,D + 38.5\,E = 3.75 \qquad (6e')$$

Using the procedure already described, the re-arrangement of the equations is shown in Table 1(*b*) and the solution in Table 3. At the end of the third set of computations, the solution shows the following rates of convergence: First column, $\frac{1.94}{13.80} = 0.14$; second column, $\frac{-1.39}{-9.30} = 0.15$; third column, $\frac{0.99}{5.17} = 0.19$; and, fourth column, $\frac{-0.31}{-1.61} = 0.19$.

The sums of the underlined quantities are given below the third set in Table 3, and the values of the unknowns are: $B = \frac{70.74}{27.75} = 2.55$; $C = \frac{-76.97}{28.8} = -2.67$; $D = \frac{39.87}{44.8} = 0.891$; and, $E = \frac{-8.69}{38.5} = -0.226$.

The values of the unknowns A and F may then be found from Equations (6*a*) and (6*f*), respectively. Substitution of the values of the unknowns in Equations (5*b*) to (5*i*), inclusive, gives the following values for the bending moments: $M_{BA} = 55.18$; $M_{BC} = -55.42$; $M_{CB} = 53.26$; $M_{CD} = -53.00$; $M_{DC} = 8.14$; $M_{DE} = -8.22$; $M_{ED} = 36.14$; and $M_{EF} = -36.12$.

The small discrepancies in the two solutions are due to slide-rule inaccuracies and to the neglect of small residuals in the calculations. The latter could have been taken into account, in the manner to be explained in connection with Example 3. Familiarity with the method of doing this enables one readily to determine that by neglecting these residuals the errors in the unknowns would be within 1 per cent.

Example 3.—The crane-girder bent shown in Fig. 3, with slight modification, is the same as that analyzed elsewhere[5] by Harold E. Wessman, M. Am. Soc. C. E. The roof truss is represented by Line $C\,C$; $B\,C$ and $B'\,C'$ are the roof columns; and $A\,B$ and $A'\,B'$ are the columns serving to support the combined roof and crane loads. The dotted line, $B\,B'$, may be considered to be a "dummy" member, with zero stiffness. The relative values of K are shown in parentheses in the diagram. In accordance with Equation (4), the bending moments are expressed by the following relations:

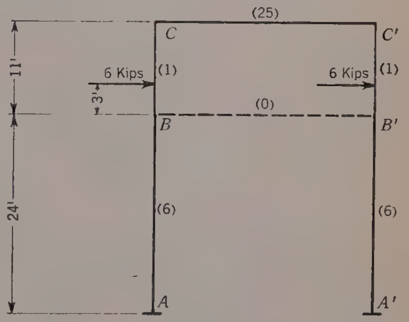

FIG. 3

$$M_{AB} = 6\,(B - \lambda_1) \qquad (7a)$$

$$M_{BA} = 6\,(2\,B - \lambda_1) \qquad (7b)$$

$$M_{BC} = 1\,(2\,B + C - \lambda_2) - 9.52 \qquad (7c)$$

$$M_{CB} = 1\,(B + 2\,C - \lambda_2) + 3.57 \qquad (7d)$$

and,

$$M_{CC'} = 25\,(3\,C) \qquad (7e)$$

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[5] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), Fig. 22, p. 49.

Equations (8) may now be written to express the necessary conditions of equilibrium in the structure:

$$14\,B + C - 6\,\lambda_1 - \lambda_2 = 9.52 \ldots\ldots\ldots\ldots (8a)$$

$$B + 77\,C - \lambda_2 = -3.57 \ldots\ldots\ldots\ldots (8b)$$

$$-6\,B + 4\,\lambda_1 = 48.00 \ldots\ldots\ldots\ldots (8c)$$

and,

$$-B - C + 0.667\,\lambda_2 = 4.02 \ldots\ldots\ldots\ldots (8d)$$

The solution of Equations (8) proceeds in the same manner as in the previous examples, four sets of calculations being required to secure a uniform rate of convergence, as shown in Table 4.

TABLE 4.—SOLUTION OF EQUATIONS (8) IN EXAMPLE 3

Line No.	(a) EQUATIONS				(b) FIRST SET				(c) SECOND SET			
	(B)	(C)	(λ_1)	(λ_2)	(B)	(C)	(λ_1)	(λ_2)	(B)	(C)	(λ_1)	(λ_2)
1.....	14	1	−6	−1	9.52	0.68	−4.08	−0.68	85.14	6.08	−36.50	−6.08
2.....	1	77	....	−1	−0.06	−4.25	....	0.06	0.01	0.88	....	−0.01
3.....	−6	....	4	....	−78.12	....	52.08	....	−54.75	....	36.50	....
4.....	−1	−1	....	0.667	−6.96	−6.96	....	4.64	9.13	−9.13	....	6.09
....	9.52	−3.57	48.00	4.02	....	....	....	....	....	....	....	....

TABLE 4.—(*Continued*)

Line No.	(d) THIRD SET				(e) FOURTH SET			
	(B)	(C)	(λ_1)	(λ_2)	(B)	(C)	(λ_1)	(λ_2)
1.....	63.87	4.57	−27.40	−4.57	47.98	3.43	−20.55	−3.43
2.....	0.05	4.56	....	−0.05	....	3.50	....	−0.05
3.....	−41.10	....	27.40	....	....	....	20.55	....
4.....	−6.93	−6.93	....	4.62	....	....	....	3.48

Upon the completion of the fourth set of calculations the rates of convergence in the separate columns are compared, to determine their degree of uniformity. They are found to be as follows: First column, $\frac{47.98}{63.87} = 0.75$; second column, $\frac{3.50}{4.56} = 0.75$; third column, $\frac{20.55}{27.40} = 0.75$; and, fourth column, $\frac{3.48}{4.62} = 0.75$.

Experience indicates that if the range of discrepancy in these ratios is within approximately 0.05, the remaining terms may generally be estimated by series summation without excessive error. If the desired agreement had not been attained at the end of the fourth set, it would have been necessary to calculate one or more additional sets.

In order to effect the summation by geometrical series, the average rate of convergence and the summation factor are required. In general, the average rate of convergence (denoted by the letter symbol, r) is best determined by dividing the sum of the underlined quantities in the last set of calculations by the sum of the corresponding quantities in the preceding set. In finding these two sums, the additions are performed without regard to sign; that is, the absolute sum is used. The summation factor is then given by the ratio, $\frac{r}{1-r}$ (the common formula for the sum of the infinite convergent series, $ar + ar^2 + ar^3 + \cdots$ being $\left.\frac{ar}{1-r}\right)$.

In Example 3 the sum of the underlined quantities in the fourth set is $47.98 + 3.50 + 20.55 + 3.48 = 75.51$; the sum of the underlined quantities in the third set is $63.87 + 4.56 + 27.40 + 4.62 = 100.45$; the average rate of convergence is $r = \frac{75.51}{100.45} = 0.75$; and the summation factor is $\frac{0.75}{0.25} = 3.00$.

The approximate sums of the quantities not included in the first four sets of calculations are then found by multiplying the underlined quantities of the final set by the summation factor, as follows: First column, $47.98 \times 3.00 = 143.94$; second column, $3.50 \times 3.00 = 10.50$; third column, $20.55 \times 3.00 = 61.65$; and, fourth column, $3.48 \times 3.00 = 10.44$.

The results of the series summations are then added to the underlined quantities that have been determined directly, and the values of the unknowns are found, as before, by dividing by the underlined coefficients in the respective equations, as shown in Table 5.

The solution of the problem is concluded by substituting the values of the unknowns in Equations (7), the following bending moments being thus found: $M_{AB} = -147.12$; $M_{BA} = 3.06$; $M_{BC} = -3.06$; $M_{CB} = -14.91$; and, $M_{CC'} = 14.70$.

TABLE 5.—DETERMINATION OF UNKNOWNS, EXAMPLE 3

Column:	First set	Second set	Third set	Fourth set	Series	Totals	Divisors	UNKNOWNS	
								Equation	Values
First. . . .	9.52	85.14	63.87	47.98	143.94	350.45	14	(B)	25.03
Second . .	−4.25	0.88	4.56	3.50	10.50	15.19	77	(C)	0.196
Third . . .	52.08	36.50	27.40	20.55	61.65	198.18	4	(λ_1)	49.55
Fourth . .	4.64	6.09	4.62	3.48	10.44	29.27	0.667	(λ_2)	43.90

In order that the relative accuracy of the method of converging increments in the present instance may be judged, the following results of a precise solution are given: $B = 25.043$; $C = 0.198$; $\lambda_1 = 49.564$; and $\lambda_2 = 43.893$.

The precision of the solution can be increased by computing the errors in the equations resulting from a first solution and using these, with reversed signs, as a new series of right-hand terms for making a few additional sets of calculations. In determining the errors in the equations at any stage of the solution, the appropriate quantities may be extended horizontally, as in the solution itself. If the utmost precision is desired, these extensions should be computed with accuracy.

In order to avoid mistakes in the solution it is desirable that the computations be checked frequently. One method of checking consists in making a careful review of each line before proceeding further. Another method makes use of an additional column of figures throughout. For the equations themselves this column contains the algebraic sum of the coefficients in a given horizontal line. In the solution, these sums are used to secure an additional extension in each line, which should agree with the algebraic sum of the remaining quantities in that line. Additions may be checked by inserting after each set a line of figures consisting of the sums of the quantities which follow the underlined item in each column of the given set; the aggregate of these sums provides a cross-check on the control column from set to set.

THE ALTERNATE SOLUTION

If the calculations resulting from a solution of a given problem by the foregoing method are recorded, step by step, on a diagram of the structure in proximity to the joints or members to which they pertain, an alternate solution possessing a number of the characteristics of the method of moment-distribution will be obtained. A detailed comparative analysis of the two solutions then enables one readily to determine the correct manner of performing the distributions, so that additional problems of the same type may be solved by use of the alternate method alone. A procedure is thus provided for extending the method of distributions to various forms of problems in which the correct distribution ratios may not be initially obvious.

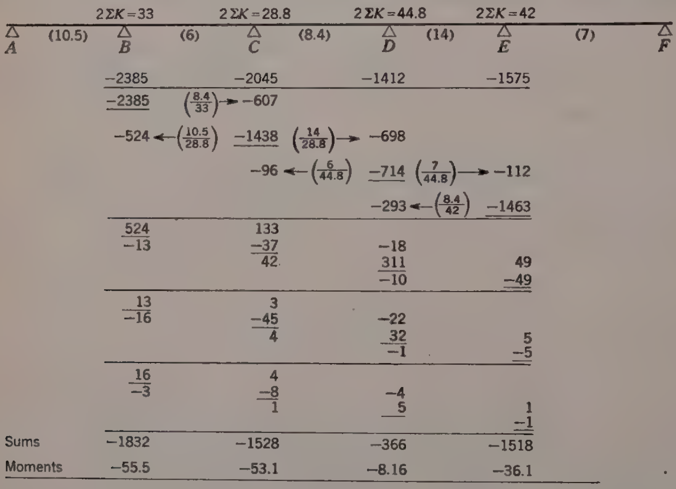

FIG. 4

In order to illustrate the foregoing statements, the solution of the equations of Example 1 is written below the various supports of the continuous beam in Fig. 4.

Comparison of the extended quantities with the coefficients of the equations in Example 1 (Table 1(*a*)) shows that the portion of the first numerical value appearing under Support *B* (− 2 385) which is to be distributed to the adjacent support depends upon the ratio of the stiffness of Span *C D* to the value of $2 \Sigma K$ at Joint *B*. Similar ratios for performing the distributions at the remaining supports are indicated in parentheses in Fig. 4. In this example, the entire process of developing the alternate solution is readily followed by referring to the original solution (Table 2), and need not be described in further detail.

In order to illustrate the method of developing the alternate solution in problems in which the unknowns of the simultaneous equations are dependent upon auxiliary equations, Example 2 will be used.

The solution is begun by writing the fixed-end moments adjacent to the various supports in Fig. 5. In accordance with Equations (6*a*) and (6*b*), and (6*e*) and (6*f*), the fixed-end moments at the supports at the two extremes of the beam are divided by − 2 and then brought over to the adjacent support. The moments at the various supports are then added, with change in sign (giving the right-hand terms of Equations (6*b*′), (6*c*), (6*d*), and (6*e*′), respectively), and the first set of calculations, as given by the original solution (Table 3), is written beneath the appropriate supports of the continuous beam in the diagram.

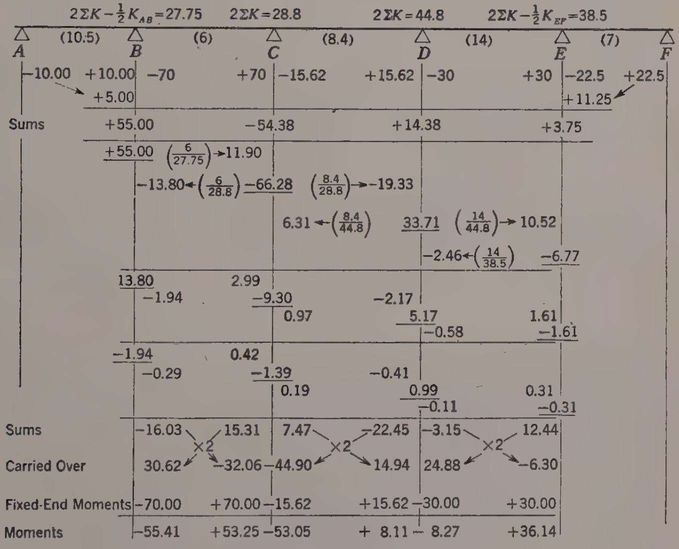

FIG. 5

In starting the analysis of the extended items, it is seen upon examination of the original solution, that the part of the first moment (+ 55.00) which goes over to the right-hand end of Span *B C* depends upon the ratio of the stiffness of that span to the quantity, 27.75. The latter quantity, according to Equations (6*a*), (6*b*), and (6*b*′), is equal to $2 \Sigma K$ at Joint *B*, less $0.5 K_{AB}$. In a similar manner, the form and significance of the remaining ratios are determined, as shown in parentheses in Fig. 5.

After the second and third sets of computations are entered on the diagram, the bending moments are ready to be calculated. This step could be done in the manner that was followed in the original solution—that is, by adding the underlined quantities, dividing by the underlined coefficients of the corresponding equations, and then substituting in the auxiliary formulas, Equations (5*c*) to (5*h*), inclusive.

However, at this point it is desirable to inquire into the significance of the extended quantities. Again referring to the original solution, it will be seen that the first extended item—namely, $11.90 \left(= \frac{6}{27.75} \times 55.00 \right)$—represents a part of the term, 6 B, in Equation (5*d*), and hence may be used in computing M_{CB}. Therefore, this quantity was written to the left of Support C. Similar reasoning governed the recording of the remaining extensions to the right or to the left of the supports, as shown in Fig. 5. Accordingly, in order to find the bending moments, the underlined quantities were ignored (and might well have been omitted from Fig. 5), and the extended items were added, giving: − 16.03, 15.31, 7.47, − 22.45, − 3.15, and 12.44 (see Fig. 5). Examination of the original solution shows that these quantities are equal to 6 C, 6 B, 8.4 D, 8.4 C, 14 E, and 14 D, respectively (in which the coefficients are the K-values of the various spans). By reference to Equations (5*c*) to (5*h*), inclusive, the remaining part of the calculation shown in Fig. 5 will be readily followed.

It is thus seen that, in problems of the type represented by Example 2, the method of converging increments not only solves the unknowns of the simultaneous equations, but actually evaluates the auxiliary equations at the same time.

CONCLUSION

Although the examples presented herein have been confined to structures involving only four equations in their analysis, the method that has been illustrated in the solution of the equations may be applied successfully to more complicated problems, provided, of course, that the analytical method on which the solution is based yields equations that are sufficiently convergent.

In many cases the rate of convergence can be improved materially by a simple re-arrangement of the equations. In determining bending moments produced by horizontal forces in multi storied structures an effective device consists in replacing each joint equation with a combined shear-and-joint equation. Doing this with Example 3, for instance, the rate of convergence is changed from 0.75 to 0.565, and the summation factor is reduced from 3.00 to 1.30.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

RECONSTRUCTION OF A PIER IN BOSTON, MASSACHUSETTS, HARBOR

By Charles M. Spofford,[1] M. Am. Soc. C. E.

Synopsis

The reconstruction of the outer parts of a large ocean pier, the wooden pile foundations of which had been seriously damaged by marine borers, is described in this paper. The wooden piles were removed and steel bearing piles many of which had a length in excess of 100 ft were substituted. These are believed to be the longest steel bearing piles ever used. They were driven by steam hammer through fill and blue clay to a bearing on bed-rock, or the hard material immediately overlying it. The piles were placed in groups of four with the upper ends protected by steel cylindrical caissons filled with concrete.

For the most part, the concrete deck of the old structure and its supporting concrete beams were removed and replaced by a new concrete deck supported on steel beams and girders. A test was made on a pile having a length of 137 ft, and the complete data of the test are given. As far as the writer is aware no other test on a pile of this length has been reported previously.

The use, in certain parts of the pier, of a light-weight concrete to protect the wooden piles from further damage is also described.

General

The control of the water-front of Massachusetts is under the direction of the Massachusetts Department of Public Works which is administered by one commissioner and two associate commissioners. Among the duties of the Department are the construction, maintenance, and operation of State piers located at various points along the Massachusetts seacoast, the largest of which is Commonwealth Pier No. 5, in South Boston.

Note.—Presented at the meeting of the Waterways Div., Boston, Mass., October 7, 1937. Written comments are invited for immediate publication; to ensure publication the last discussion should be submitted by **December 15, 1938.**

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Boston is one of the important ports in the United States, its standing on the Atlantic seaboard being exceeded (as measured by the value of its foreign and domestic commerce) only by the Ports of New York, N. Y., Philadelphia, Pa., and Baltimore, Md. Commonwealth Pier No. 5, with its transit sheds and other facilities, is the principal pier for passenger ships on the southerly side of the harbor and is used by several foreign lines, including the Italian, French, and Holland-American Lines. It is also used by the American Hawaiian and the Luckenbach Lines furnishing service to the Pacific Coast, and by the Canadian National Steamship Company operating to the West Indies. Much freight is received at Pier No. 5, wool being one of the principal products. Boston is the second largest wool port of the world, exceeded only by the Port of London, England.

The pier provides berths in excess of 1 100 ft in length, with a depth of water of at least 40 ft at mean low tide. It is sufficient in size and draft to accommodate the largest ships now in existence. Fig. 1 shows the location of this and other State piers in Boston.

HISTORICAL

The construction of the pier was begun in 1897, under the direction of the State Board of Harbor and Land Commissioners, by the building of a granite sea-wall around a site, 300 ft wide by 1 150 ft long, and the filling of the interior with material dredged from docking spaces alongside the pier. This wall was supported on spruce piles capped with spruce planks and timber. The spaces between the piles and in front of them were filled with small stone chips and gravel, except that the face of the slope in front of the wall was protected by large stones weighing 100 to 300 lb each. After the solid part of the pier had been completed, it was surrounded by a 50-ft platform of hard pine supported on oak piles. The heads of the piles were covered with coal tar applied at a temperature of not less than 120° F, but no other preservative treatment was used. The total cost of this part of the pier was not quite $400 000.

The then comparative inaccessibility of the pier by land delayed the making of satisfactory arrangements for its use. In July, 1907, a part of it was seriously damaged by fire. The pier was finally leased on November 1, 1910, together with certain land in its rear, to the Old Colony Railroad Company at the annual rate of $75 000, the lessee to pay the city taxes. This lease was not of long duration as on June 28, 1911, the Massachusetts Legislature created a board entitled "The Directors of the Port of Boston" and transferred to this board all the rights, powers, and duties of the former Board of Harbor and Land Commissioners pertaining to the Port of Boston. It also authorized the expenditure of $9 000 000. The Directors decided to build a terminal to serve large vessels and reached the conclusion that Commonwealth Pier No. 5 afforded the best opportunity for the development of such a terminal.

After months of negotiations the Directors secured from the Old Colony Railroad Company, on November 6, 1912, the control of the pier and at once set aside $2 500 000 for improvement, and work was started two days later. This improvement included dredging the berths alongside to 40 ft below mean low water; cutting off burned piles on the easterly side to Elevation + 10

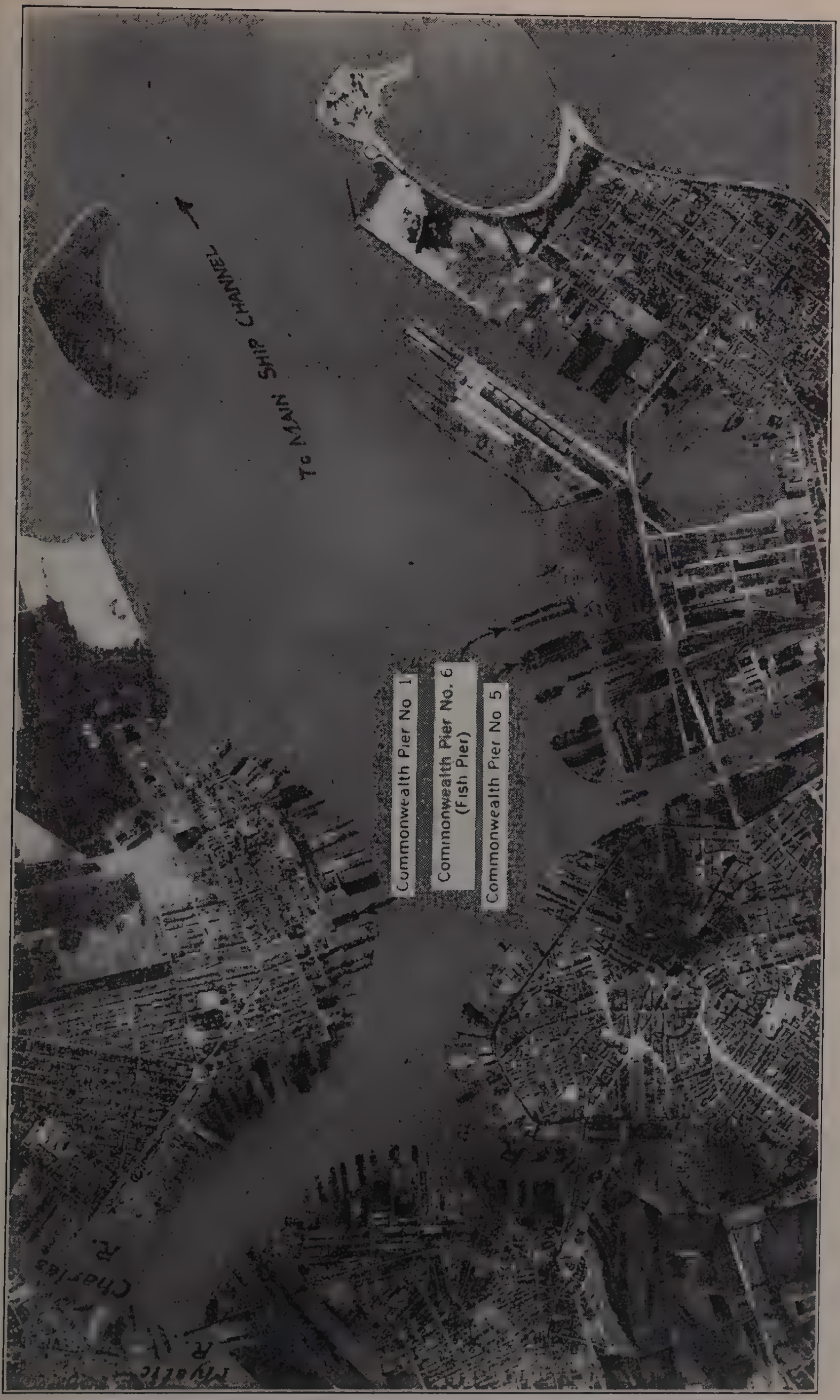

FIG. 1.—AIRPLANE VIEW OF BOSTON HARBOR, SHOWING COMMONWEALTH PIERS

and the remainder of the piles to Elevation + 14, and supporting the deck structure by concrete beams resting on the original oak piles; replacing the timber platform with a concrete deck, covering the entire pier except the outer parts of the platforms, and the outer end, with two-story non-combustible sheds supplied with facilities for handling both freight and passengers; and constructing railroad and highway connections. The pier was opened for business on May 31, 1913, when a new service of the Hamburg-American Line was instituted, the western half of the pier being assigned to this line. The Directors also agreed to furnish the White Star Line with service on the east side.

Pier improvements, as originally planned, were estimated to cost $1 250 000, but through amplifications as a pier and as a monument to advertise Boston the total sum finally expended was approximately $4 000 000.

ATTACKS BY MARINE BORERS

In 1925, the attention of the Public Works Department was called to the fact that marine borers were becoming active in Boston Harbor, and it was decided to investigate a few of the foundation piles under the platforms at Commonwealth Pier No. 5 to determine the extent of the destruction by these borers. Later examinations were made in 1932 and in 1934, and the results of all these examinations are shown in Table 1.

TABLE 1.—EFFECT OF MARINE BORERS, COMMONWEALTH PIER NO. 5

Condition	(*a*) 1925 Investigation				(*b*) 1932 Investigation					(*c*) 1934 Investigation					
	East side	West side	Total*	Percentage of total	East side	West side	North end	Total*	Percentage of total	East side	West side	North end	Clusters	Total*	Percentage of total
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
One-eighth gone	1	0	1	0.9	16	26	9	51	10.5	56	53	10	13	132	13.9
One-fourth gone	0	9	9	7.6	34	46	19	99	20.4	46	103	37	24	210	22.2
One-half gone	2	0	2	1.7	12	60	22	94	19.4	7	38	20	4	69	7.3
Three-fourths gone	..	..	..	..	4	5	4	13	2.7	9	15	2	0	26	2.7
Slightly touched	28	25	53	44.9	91	76	15	182	37.6	175	114	12	134	435	46.0
Uninjured	23	30	53	44.9	25	18	2	45	9.4	38	25	7	5	75	7.9
Total	54	64	118	100.0	182	231	71	484	100.0	331	348	88	180	947	100.0

* Number of piles.

It will be seen that, in the 1925 examination, approximately 55% of the piles examined were found to be affected by marine borers, whereas in the last examination about 92% were affected. The action of the borers had been constantly increasing and, based on the results of the last examination, about 10% of the piles were one-half or more destroyed. During the last examination (Table 1(*c*)), six of the piles were found to have a diameter of not more than 3 in. at the mud line, instead of their original diameter of about 10 in.

As a result of these examinations, the Engineers of the Department recommended that studies be made of means to prevent further action by marine borers and a tentative plan for placing solid filling under the pier platforms, retained by a steel bulkhead around the pier, was developed at an estimated cost of approximately $437 000. This method had to be rejected after the nature of the subsoil was thoroughly investigated.

It is interesting to note that during reconstruction most of the piles that were examined by the divers were pulled out by the contractor, and it was possible to get a complete view of the damage. In general, the reports of the divers were found to be correct except that piles reported three-fourths gone were found to be in a worse condition than reported. Piles upon which the original bark remained in place were found to be in good condition, but wherever the bark was missing, the pile had been badly attacked by the worms. The major part of the damage was caused by *Limnoria;* some *Teredo* were found, but not in sufficient number to affect the piling seriously.

Methods of Reconstruction

The reconstruction may be treated conveniently under two divisions: (1) The repairs of the platforms and pile foundations of the outer parts of the pier; and (2) the repairs of the foundations of the head-house at the shoreward end of the pier.

Platforms and Pile Foundations.—Three different types of bulkhead construction were considered for this work: (*a*) An enclosure of steel sheet-piles filled with sand and gravel; (*b*) built-up "master-piles" with intermediate sheet-piles; and (*c*) the cellular type. Type (*a*) consists of driving a row of steel sheet-piles immediately outside the present face of the deck, holding the pile tops back by wales and tie-rods, with sand and gravel fill around the timber piles, thus preventing further damage to them from marine borers. With the data available from the borings and laboratory analyses described elsewhere, it was possible to make studies to determine the size and length of the steel sheet-piles necessary to withstand the pressure from the fill to be placed around them to the elevation necessary to protect them from further damage from marine borers. From these studies, it was found that due to the soft underlying clay, the piles would have to be driven to an excessive length, in order to obtain sufficient pressure at the toe to hold them in place. This was determined by computing the active earth pressure on the bulkhead by the application of the Rankine equations and equating this pressure with the resistance of the blue clay at the face of the pile corresponding to its shearing resistance. The latter was determined by tests on undisturbed samples.

The width of the wharf platform was too small to permit the sheeting to be tied back efficiently to the cross-bents supporting the wharf and the only feasible method of restraining the upper ends of the sheeting seemed to be to build a series of concrete "dead men" at a considerable distance back of the sea-wall and to tie the ends of the sheeting to these "dead men" by heavy steel tie-rods. This process would have necessitated the drilling of holes in the existing walls, a laborious, time-consuming, and expensive process.

Type (*b*) consists of the construction of "master piles" composed of heavy built-up sections placed at intervals along the front of the wharf face, and the

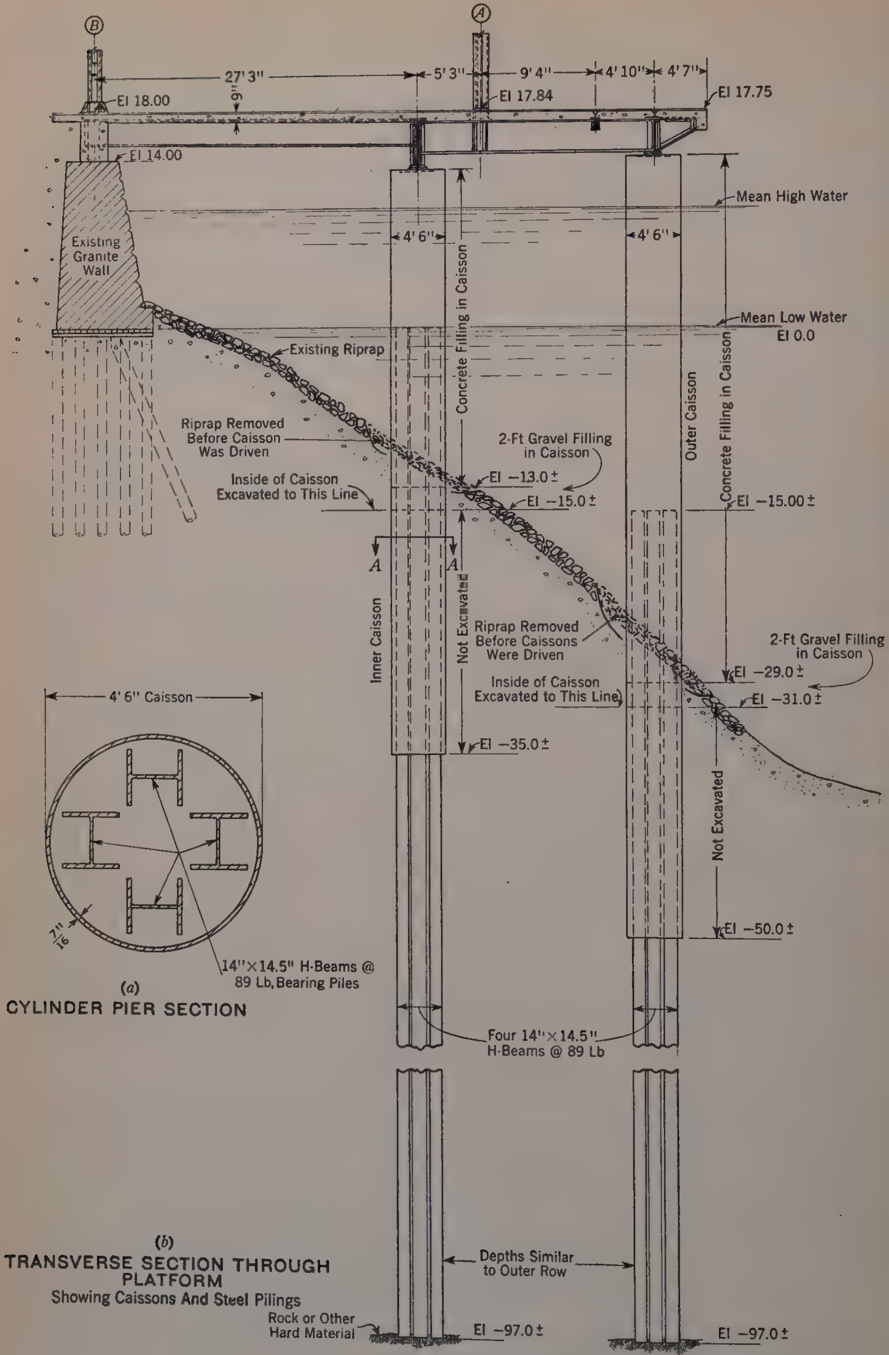
B
A
27' 3"
5' 3"
9' 4"
4' 10"
4' 7"
El 18.00
9"
El 17.84
El 17.75
El 14.00
Mean High Water
Existing Granite Wall
4' 6"
4' 6"
Concrete Filling in Caisson
Mean Low Water El 0.0
Existing Riprap
Outer Caisson
Concrete Filling in Caisson
Riprap Removed Before Caisson Was Driven
2-Ft Gravel Filling in Caisson
El −13.0 ±
Inside of Caisson Excavated to This Line
El −15.0 ±
El −15.00 ±
A
A
Inner Caisson
Not Excavated
Riprap Removed Before Caissons Were Driven
2-Ft Gravel Filling in Caisson
El −29.0 ±
Inside of Caisson Excavated to This Line
El −31.0 ±
El −35.0 ±
Not Excavated
El −50.0 ±
4' 6" Caisson
7/16"
14"×14.5" H-Beams @ 89 Lb, Bearing Piles
(a)
CYLINDER PIER SECTION
Four 14"×14.5" H-Beams @ 89 Lb
(b)
TRANSVERSE SECTION THROUGH PLATFORM
Showing Caissons And Steel Pilings
Depths Similar to Outer Row
Rock or Other Hard Material
El −97.0 ±
El −97.0 ±

Fig. 2.—Transverse Section of Platform, Showing Caissons and Steel Piling

driving of intermediate sheet-piles between, and supported by, these built-up members. For this type of construction the master piles would have to be held at the bottom by being driven a considerable distance into the underlying clay and at the top by tie-rods fastened to concrete "dead men" as proposed for Type (*a*). The same difficulties were found in supporting this type of bulkhead at its toe as were found for the simple steel sheet-piles.

The third type (Type (*c*)) of bulkhead considered was a cellular type but, among other difficulties, it was found that the available distance between the sea-wall and the face of the existing wharf platform was insufficient to provide the necessary stability for this type of construction which would have to be built on a steep slope.

In view of the foregoing difficulties with bulkhead construction and its excessive cost, it became necessary to develop another method for this part of the work. After considerable study it was decided to use steel bearing piles driven either to bed-rock or into the hard overlying strata and to support the reinforced concrete slab by a framework of steel beams.

The following description covers this method of construction: Steel bearing piles, 14 in. by 14.5 in., weighing 89 lb per ft were driven, in clusters of four, to practical refusal into bed-rock or into hard overlying material. During driving, they were held in position laterally by timber frames. The upper parts of each group were surrounded by a structural steel cylinder filled with concrete, and concrete was deposited in the caisson above the tops of the steel piles which, in no case, were above mean low water. The piles and caissons are shown in Fig. 2.[2] For most of the work the cylinder piers thus formed were spaced 40 ft, center to center, longitudinally and 21 ft 5 in., center to center, transversely. The various steps in the construction were as follows:

(1) The rip-rap and a portion of the silt in the area in which a group of piles was to be driven were excavated to as great a depth as feasible;

(2) The steel caisson for the group of piles was generally driven by a steam hammer to a minimum depth of 5 ft in the clay subsoil with its top at the proposed level of the bottom of the new steel and concrete deck;

(3) The material within the caisson was excavated to a depth of 15 ft below the anticipated level of the tops of the piles;

(4) The four steel piles in each caisson were then driven, one by one, by a steam hammer. (Except in a few cases, all piles had to be spliced once. The minimum length of pile was 98 ft. In certain cases the piles twisted in driving and in certain cases the driving of one pile carried down another pile. Possibly these effects would have been avoided by increasing the spacing, but this would have increased the cost and added to the load carried by individual piles);

(5) A 2-ft layer of sand and gravel was deposited within the caisson around the piles and over the overlying soil to form the bottom for the concrete to be placed in the caisson; and,

(6) Concrete was placed in the caisson around the steel piles by means of a chute, the caisson proving to be water-tight, so that it was unnecessary to place an under-water concrete seal at the bottom.

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[2] See, also, *Civil Engineering*, December, 1937, p. 843.

Prior to driving the piles, borings were made to determine the required lengths. A tolerance of 6 in. in the location of the center of the caisson caps was permitted, and no caisson moved more than this distance from its correct position.

In the process of construction, it was necessary to cut holes in the roof and second floor of the pier shed in order to place the piles and the steel cylinders in position. In driving, the tops of the piles were protected, somewhat ineffectually, by various types of cushions. Some of the rivets in the splices were loosened during driving. However, the ends of the abutting sections at the splices were faced and enough rivets used to insure that the abutting piles would be held in line. Fig. 3 shows the splices used. Each pile was designed for 100 tons and, as seen in Fig. 3, thirty-six shop rivets were used above the splice and forty-four field rivets below it. In the worst case, 25% of the shop rivets and 32% of the field rivets were loosened in the lowermost splice during the first stage of the driving. Assuming that further loosening of the rivets in this splice would occur in the second (final) stage of the driving in proportion to those already loosened, twenty tight shop rivets and twenty tight field rivets would still be left. This number is more than ample to comply with the specifications of the American Railway Engineering Association for splicing bridge chords.

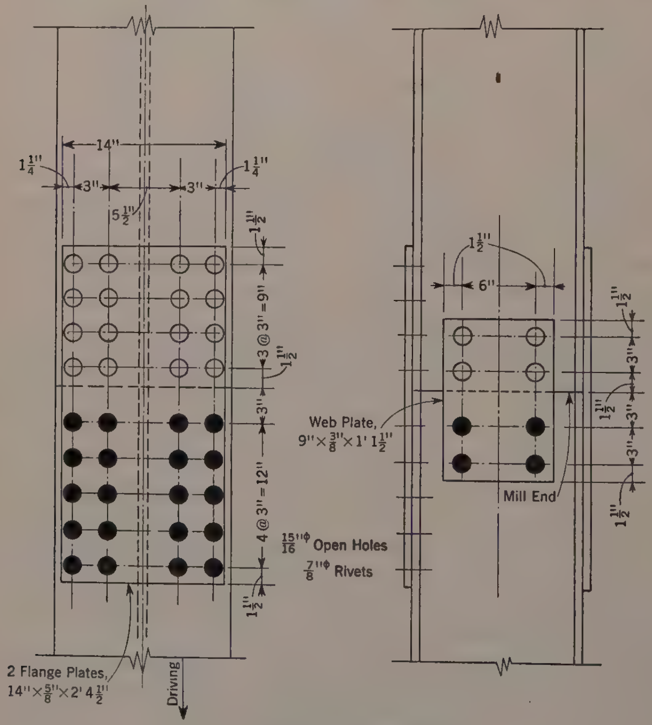

FIG. 3.—SPLICES IN STEEL PILES

After the cylinder piers were constructed, the platform shown in Fig. 4 was built. The wharf-shed columns were supported temporarily and the steel framework forming the support for the concrete deck was installed. The steel of this framework consisted chiefly of 30-in. and 36-in. **H**-beams of silicon steel placed longitudinally, and supporting transverse **H**-beams, also of silicon steel. The longitudinal members were usually continuous over two spans. Transverse expansion joints were provided at 80-ft intervals.

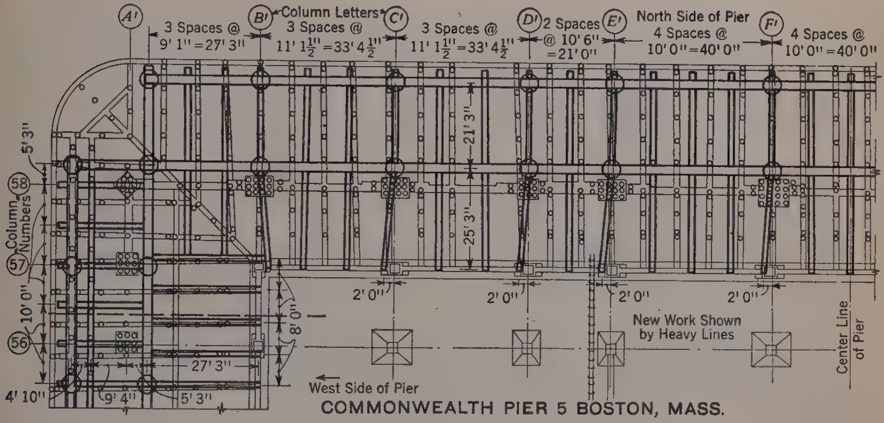
A'
B'
Column Letters
C'
D'
E'
F'
3 Spaces @ 9' 1" = 27' 3"
3 Spaces @ 11' 1½" = 33' 4½"
3 Spaces @ 11' 1½" = 33' 4½"
2 Spaces @ 10' 6" = 21' 0"
North Side of Pier
4 Spaces @ 10' 0" = 40' 0"
4 Spaces @ 10' 0" = 40' 0"
5' 3"
58
Column Numbers
57
10' 0"
56
4' 10"
9' 4"
27' 3"
5' 3"
21' 3"
25' 3"
8' 0"
2' 0"
2' 0"
2' 0"
2' 0"
New Work Shown by Heavy Lines
Center Line of Pier
West Side of Pier
COMMONWEALTH PIER 5 BOSTON, MASS.

FIG. 4.—ARRANGEMENT OF STEEL BEAMS IN PLATFORM (SYMMETRICAL ABOUT CENTER LINE)

Head-House Foundations.—The method of reconstructing the wharf platform could not be used at the shoreward end of the pier under the head-house (which is a massive brick building extending out to within 20 ft of the face of the wharf platform), on account of the difficulty in supporting, temporarily, the heavy columns and brick walls of the head-house structure. Various methods were proposed for this part of the work and that finally adopted consisted of driving, to a limited depth, a light-weight steel sheet-pile bulkhead just outside the foundations of the building walls and filling the space between it and the existing sea-wall, to approximately low-water level, with a light-weight concrete. This concrete was placed in comparatively small quantities so that it would exert a minimum pressure against the steel bulkhead.

It was originally proposed to use fine and coarse coke aggregate for the concrete, and experiments were made to determine the feasibility of placing this class of concrete under water. These tests, on a small scale, were successful; but, when the coke concrete was deposited in large quantities through tremies, it was found that there was a tendency for the large pieces of coke to float. Therefore, cinders were substituted for the coke and the piles were protected up to low water back of the bulkhead by placing this concrete in place with a tremie. To protect the part of the piles above low water up to the bottom of the beam and column footings, forms were built around the pile clusters and on either side of the pile bents, using creosoted timber, and the spaces within these forms were filled with the cinder concrete. In this manner, it was possible to provide an efficient protection of all piles within the building site itself without the necessity of removing any of the structure. The adequacy of this method of protection was referred to William F. Clapp, Consulting Biologist, who reported as follows:

> "(1) Effect on Marine Borers Now Present. All destructive marine organisms require a constant flow of salt water containing a sufficient amount of oxygen to support life in the case of *Limnoria* and *Chelura*, and to supply food as well in the case of *Teredo*. The proposed protection would completely prevent the flow of salt water to the protected piles. This will result in the immediate extermination of all species of marine borers on or in the protected wood. There will be therefore no possibility of further pile deterioration from the marine borers now present. This has been demonstrated in numerous instances where protection of similar type has been used.
>
> "(2) Efficacy against Future Attack. Since the protection shown on the plans effectually prevents the free and continuous flow of salt water to the protected parts, under these conditions, no future invasion by marine borers of any type can take place. Consequently, in my opinion, the piles thus protected will be thoroughly safeguarded against future attack by any species of marine borer."

The platforms extending from the building outward to the face of the dock were removed, and new creosoted piles were driven to support a new concrete deck. The dock opposite the head-house had not been dredged to the full depth required alongside the pier structure itself, and when the platform piles were pulled in this part of the pier, it was found that the damage to them by marine borers had been less than farther out into the harbor where the water was materially deeper.

Subsoil Investigations

Preliminary investigations of the subsoil conditions were made by the usual wash-boring methods. Undisturbed samples of the clay were also obtained by a method developed under the supervision of Glennon Gilboy, Assoc. M. Am. Soc. C. E., in the Soil Mechanics Laboratory of the Massachusetts Institute of Technology, at Cambridge, Mass., in co-operation with the former Committee of the Society on Earths and Foundations. This process consists of driving a 6-in. casing into the soil, pumping and bailing out the water, and removing the soil by an auger to the elevation where the sample is to be secured.

A 2-in. pipe, enlarged to a 4-in. spoon, is then lowered through the casing into the soil and the sample cut off at the bottom of the spoon by a piano wire which extends to the top of the pipe. A vacuum is applied to the 2-in. pipe while raising, to help hold the sample in place. After it was finally decided to use steel piles driven to bed-rock, a series of core-borings was also made. The underlying bed-rock was found to consist of seamy slate rock varying, in elevation referred to mean low tide, from − 94.00 to − 142.67. Core-borings were also made to determine the elevation and character of the bed-rock.

Pile Test

In view of the unusual length of the steel piles under consideration for the construction of the platforms and pier foundations, a test was made on one pile, consisting of a 14-in. by 14.5-in. by 89-lb steel bearing pile having a cross-section area of 26.19 sq in. and a total length of 137 ft and consisting of three sections which were spliced in the field during driving. The pile was driven through overlying materials, mostly blue clay, with a steam hammer, and to refusal on bed-rock. Refusal occurred 113 ft below mean low water, and ten blows were struck by the hammer without further penetration. At the conclusion of the test 54 ft of the pile was unsupported and 83 ft was in the blue clay. A steel loading platform and steel rails were used for the applied load, as shown in Fig. 5. The basic data applying to the test were as follows:

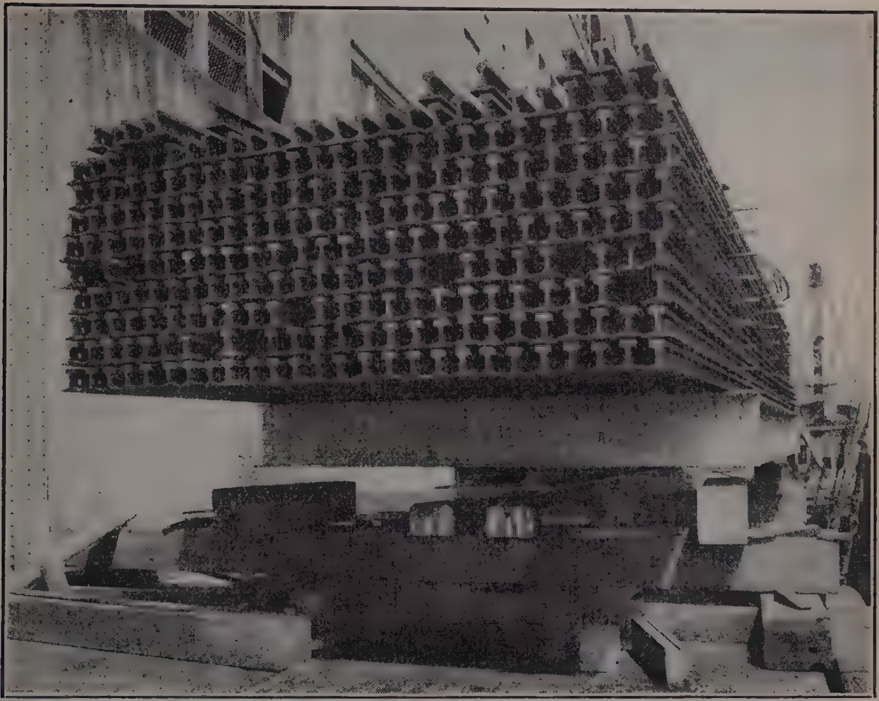

FIG. 5.—LOADING FOR TEST PILE

Description	Loads, in pounds
Weight of platform	10 552
Weight of cap	2 674
244 rails @ 880	214 720
Weight of hammer	14 000
Total weight (120.97 tons)	241 946

The deformation was measured at a point approximately 3 ft below the top of the pile and was observed from an initial load of approximately 40 tons to a maximum load of 121 tons at intervals of about 10 tons. The maximum load was allowed to remain on the pile for 24 hr, the total schedule being as follows:

Began loading April 14, 1936; loaded 63.6 tons; finished loading April 15, 1936, 11:30 A.M.; total load, 121 tons; began unloading April 16, 1936; 11:40 A.M.; and finished unloading April 16; 3:45 P.M. During unloading, observations were taken at intervals of about 10 tons from 86 tons until the pile was completely unloaded. The log of the pile-driving is given in Table 2 and a diagram of the deformation of the pile for various loads during loading and unloading is shown in Fig. 6 which also shows lines giving the theoretical deformation for lengths of 51 ft and 134 ft assuming the full load to be distributed over these respective lengths—that is, assuming no load to be transferred to the surrounding material.

TABLE 2.—LOADING LOG, INDIVIDUAL PILE TEST

(a) LOADING			(b) UNLOADING		
Weight, in Tons		Deformation, in feet	Weight, in Tons		Deformation, in feet
From	To		From	To	
0	40.8	0.002	121.0	86.4	0.004
40.8	52.2	0.003	86.4	75.0	0.005
52.2	63.6	0.005	75.0	63.6	0.004
63.6	75.0	0.003	63.6	52.2	0.003
75.0	86.4	0.006	52.2	40.8	0.002
86.4	97.8	0.003	40.8	29.4	0.002
97.8	109.2	0.002	29.4	18.0	0.003
109.2	121.0	0.003	18.0	0	0.002
Total....	....	0.027	....	....	0.025

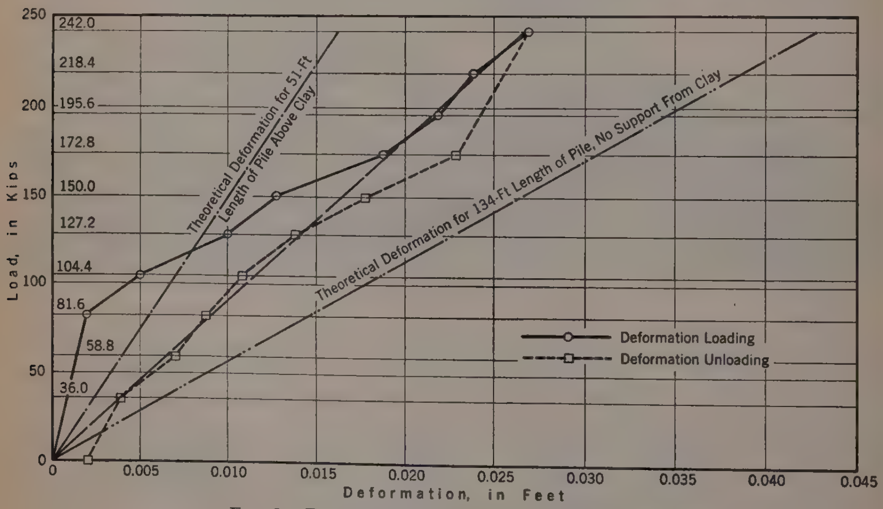

FIG. 6.—DEFORMATION DIAGRAM FOR TEST PILE

It will be noted that the deformation up to the load of 81 600 lb equals the permanent set of 0.002 ft as shown by the unloading diagram, this probably being equal to the penetration of the pile into bed-rock. From 81 600 lb to

195 600 lb, the deformation agrees very closely with that which would occur for the total length of 134 ft, which would seem to indicate that deformation was taking place over the full length of the pile. From a load of 195 600 to the full load of 242 000 lb, the deformation at all points was less than the theoretical deformation for a 134-ft length, indicating that during this period the load was being carried partly by the clay and partly on the rock.

When the pile was unloaded, the part above the clay first expanded at the theoretical rate for a 51-ft length of pile, the clay apparently holding the lower part of the pile firmly in position. From a load of 172 800 lb to a load of 127 200 lb the deformation of the pile was approximately equal to that for its entire length, namely, 134 ft. From the latter load to a load of 36 000 lb the pile expanded at a rate intermediate between that which would have occurred had the load been applied over its entire length and that which would have occurred had it been applied over only 51 ft of pile. It is impossible to discern from the diagram how much of the load was carried by the clay and how much directly on rock, but in the judgment of the writer, a considerable part of the load was carried by the clay.

Special Contract Requirements

Substitution of Steel Cylinders for Steel Piles.—After the contract for the work on the outer section of the pier had been signed and the work begun, the contractor requested permission to substitute steel cylinders driven into bed-rock by rotation for the steel piles included in the contract, and proposed a considerable reduction in the contract price if this change was approved. After careful consideration by the State Engineers, the Engineers of the Public Works Administration and the Consulting Engineers, permission was given for this change provided two test caissons could be driven successfully. Thereupon, the contractor undertook the construction of a test caisson. The experiment was not successful, however, as the material overlying the clay, consisting in part of rip-rap, offered such resistance that the trial cylinder was irrevocably damaged long before it reached bed-rock. Further efforts along this line were not made, and the construction was completed in the manner originally contemplated and described in this paper.

Removal of Existing Piles and Bracing.—The conditions of the contract required the eventual removal, in whole or in part, of all main piles, spur shores, and cross-bracing under the existing platform. Piles that interfered with the driving of steel piles or caissons were pulled, and other piles were cut off just above the ground surface. The existing fender piles and capping and the pile and timber corners were also removed and replaced by new fenders.

Avoidance of Interference with Business of Pier.—In order to avoid interruption of business at the pier as far as was reasonably possible, the pier was divided into five sections; the contractor was allotted one section at a time and was required to complete the work in that section before proceeding with work in another unless the pile and caisson work in another section could be done simultaneously without interfering with the business at the pier.

Settlement of Piles.—No settlement, either in the piles or in the concrete deck, has been discovered since the completion of the pier.

Bid Prices.—The following are some of the unit prices offered by the low bidder:

Caissons for the inner row, in place, per caisson..	$ 790.00
Caissons for the outer row, in place, per caisson..	1 050.00
Steel piles per pound........................	0.037
Structural steel per pound....................	0.050
Concrete per cubic yard......................	18.00
Yellow pine timber creosoted per 1 000 B. M....	120.00

CONCLUSIONS

The method of construction described in this paper, which involves the use of long steel piles driven to bed-rock and supporting a steel and concrete deck, is satisfactory for conditions such as existed at this pier. Furthermore, the use of a light-weight cinder concrete surrounding the wooden piles is a satisfactory method of protecting such piles from further damage by marine borers.

ACKNOWLEDGMENTS

The thanks of the writer are due to the Hon. William F. Callahan, Commissioner, Richard K. Hale, M. Am. Soc. C. E., Associate Commissioner, G. H. Delano, Chief Engineer, John W. Ferguson, M. Am. Soc. C. E., District Engineer in Charge of the Boston District, and Charles J. O'Donnell, Assistant Civil Engineer, Massachusetts Department of Public Works. These officials co-operated generously in furnishing information and giving other assistance in the preparation of the paper. John Ayer, M. Am. Soc. C. E., with Mr. O'Donnell, assisted in writing it. Fig. 1 was supplied by the United States Engineer Department (Boston, Mass., District); Glennon Gilboy, Assoc. M. Am. Soc. C. E., made the tests on undisturbed soil samples in the Soil Mechanics Laboratory of the Massachusetts Institute of Technology, and, the description of the single pile test presented herein was written by B. A. Rich, M. Am. Soc. C. E.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

MECHANICAL STRUCTURAL ANALYSIS BY THE MOMENT INDICATOR

BY ARTHUR C. RUGE,[1] ASSOC. M. AM. SOC. C. E.,
AND ERNST O. SCHMIDT,[2] ESQ.

SYNOPSIS

A new method for the mechanical analysis of models of indeterminate structures is described. This method involves the use of a simple and inexpensive instrument known as the "moment indicator." A suitable technique for the application of the method to models made of semi-plastic materials like celluloid is described, and the results of measurements on a Vierendeel structure are given. Other applications are suggested.

The writers conclude that the technique discussed is capable of giving results accurate to 1%, and that an accuracy of 2% or 3% is easily obtained with considerable economy of time over other available methods of stress determination. It is suggested that the technique is suitable for use in practical engineering offices as a design tool.

INTRODUCTION

The purpose of the paper is to describe a simple and direct method of determining the bending moments in models of structural frames due to external loads. The method is particularly well adapted to the study of structures having a high degree of statical indeterminancy, whether the loading is static or dynamic in character.

For statical load conditions, the moment indicator is a simple and inexpensive instrument, capable of yielding very accurate results. The technique is of such simplicity and speed that it could well be adapted for routine design work in engineering offices. Less manipulative skill is required than for other available methods of mechanical analysis capable of giving comparable accuracy.

As to the merits of the model method for the analysis of complex structural problems, very little needs to be said beyond making reference to the excellent

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NOTE.—Written comments are invited for immediate publication; to ensure publication, the last discussion should be submitted **December 15, 1938.**

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[2] Asst., Dept. of Civ. and San. Eng., Mass. Inst. Tech., Cambridge, Mass.

paper by R. L. Templin, M. Am. Soc. C. E., published in 1937.[3] Several effective techniques have been developed for determining the stresses in structural models, the outstanding one being the deformeter method introduced by George E. Beggs, M. Am. Soc. C. E.[4]

No attempt is made herein to compare one model method with another, since each has its own particular applications for which it is especially suited. Indeed, the efficiency of any method is too closely associated with the skill and preferences of the investigator to make general comparisons possible. In so far as correctness of results is concerned, there is little choice between methods known to be based upon sound principles of mechanics.

After giving attention to the principle of the moment indicator, the writers confine themselves primarily to a description of its application to the particular case of celluloid models of two-dimensional indeterminate frameworks. The behavior of celluloid as a model material is discussed, and an adequate technique for dealing with it and with similar semi-plastic materials is described. Results of measurements and calculations for a four-panel Vierendeel structure are given in sufficient detail to establish the probable accuracy of the moment-indicator method. Other applications of the method are discussed briefly.

PRINCIPLE OF THE MOMENT INDICATOR

Consider a straight bar, subjected to bending moments and shear, and having no external loads, on a uniform section of length, L (see Fig. 1(a)). The moment indicator consists of a set of rigid levers attached at the ends of this section. The levers are provided with indices at one-third and two-thirds the length, L.

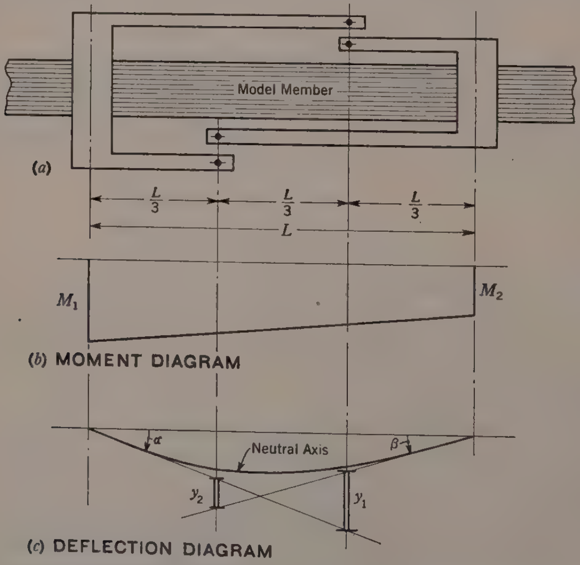

FIG. 1.—DIAGRAM ILLUSTRATING PRINCIPLE OF MOMENT INDICATOR

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[3] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 1211.

[4] *Loc. cit.*, Vol. 88 (1925), p. 1208.

In Fig. 1(*c*), Deflections y_1 and y_2 represent the relative motions of the indices due to the applied bending moments of Fig. 1(*b*). The relation of y to M is easily found: $\alpha = \frac{M_1 L}{3 E I} + \frac{M_2 L}{6 E I}$, and $\beta = \frac{M_1 L}{6 E I} + \frac{M_2 L}{3 E I}$. Therefore:

$$y_1 = \frac{2}{3} L \alpha - \frac{1}{3} L \beta = \frac{M_1 L^2}{6 E I} \quad \text{(1)}$$

The measurement of y_1 and y_2 thus gives the bending moments directly:

$$M_1 = \frac{6 E I}{L^2} y_1 \quad \text{(2a)}$$

and,

$$M_2 = \frac{6 E I}{L^2} y_2 \quad \text{(2b)}$$

At either point, motion of the indices toward each other corresponds to a positive bending moment. (In Fig. 1, M_1, M_2, α, β, y_1, and y_2, are shown positive.)

Equations (2) hold only if the shear deflections can be neglected. In short and deep bars the influence of shear must be taken into account, the correction to y being deduced as follows: The shear in the section equals,

$$s_s = \frac{M_1 - M_2}{L} \quad \text{(3)}$$

The contribution of shear deflection to indicator reading is,

$$\Delta y_1 = -\Delta y_2 = \frac{s_s}{G A'} \quad \text{(4)}$$

in which G is the shear modulus of the material, and A' is the effective area of cross-section in shear. Since Δy will be a small percentage of y (rarely more than 5%), it is sufficient to introduce the approximate values of M from Equations (2) into Equation (3); and to substitute for G, its value, $\frac{E}{2\left(1 + \frac{1}{m}\right)}$, in which m = Poisson's number. Then, Equation (4) gives:

$$\Delta y_1 = \frac{12 I}{L^2 A'} \left(1 + \frac{1}{m}\right) (y_1 - y_2) \quad \text{(5)}$$

In the case of a bar of rectangular cross-section having a depth, d, $A' = \frac{A}{1.2}$ (approximately). Furthermore, for celluloid (and many other model materials), $\frac{1}{m} = 0.4$. There result, then, the corrections:

$$\Delta y_1 = 1.68 \left(\frac{d}{L}\right)^2 (y_1 - y_2) \quad \text{(6a)}$$

and,

$$\Delta y_2 = 1.68 \left(\frac{d}{L}\right)^2 (y_2 - y_1) \tag{6b}$$

Therefore, the true bending moments are:

$$M_1 = \frac{6\,E\,I}{L^2} (y_1 - \Delta y_1) \tag{7a}$$

and,

$$M_2 = \frac{6\,E\,I}{L^2} (y_2 - \Delta y_2) \tag{7b}$$

In many practical design problems, the correction represents an unnecessary refinement. It is so easily calculated, however, that it may as well be taken into account.

By obvious extension of the foregoing analysis, the effect of loads applied within Section *L* can be included. Curved bars and bars of non-uniform cross-section can likewise be treated by determining the appropriate slope-deflection coefficients, either experimentally or by calculation. In nearly all haunched-beam problems the moment indicator can be used on the straight uniform section between haunches. In certain special cases the indices may be placed profitably at points other than the third points, but the arrangement shown in Fig. 1 will be found most satisfactory for general use.

Description of the Instrument

In the practical design of the indicator, it is advantageous to deviate from the arrangement shown schematically in Fig. 1 to the extent of placing all the indicating levers on the same side of the bar. Fig. 2 illustrates a construction that was found very convenient and satisfactory for problems involving static loads. Except as shown, the parts are of aluminum.

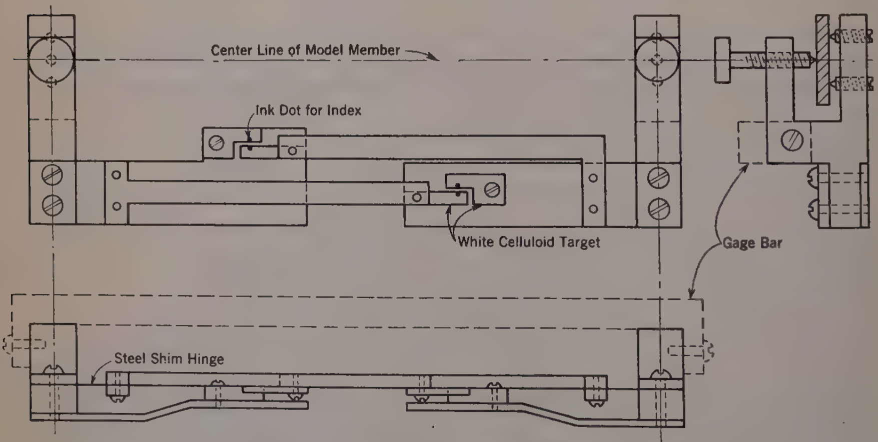


Fig. 2.—Detail of Moment Indicator

The indicator is fixed on the model bar by two clamps (see Fig. 3), each provided with three sharply pointed screws. This clamping arrangement has the important advantage that no restraint is imposed upon the elastic deflections of the bar. The long levers are attached to the clamps by means of thin spring hinges, their free ends sliding on an extension of the short levers in order to keep the indices always in the same focal plane for microscope reading. It will be seen that the levers are placed so that the indices are approximately in the middle plane of the model bar; thus, twisting of the bar will not produce error in the reading.

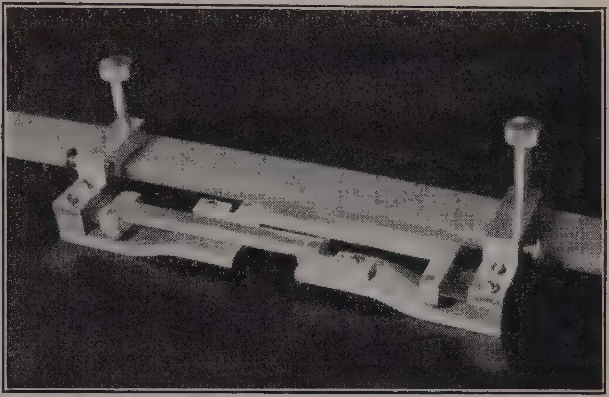


FIG. 3.—MOMENT INDICATOR MOUNTED ON BAR: GAGE REMOVED

The indices are small India ink dots on white celluloid. A gage bar, shown dotted in Fig. 2, conveniently sets the exact positions of the clamps and indices, the gage being removed after the clamp screws are tightened.

It will be evident to the reader that many interesting variations in design are possible. For example, one instrument was built with a gage length variable from 4 to 12 in. Furthermore, the sensitivity may be increased ten or more times by means of a magnifying lever, or by the use of measuring devices more sensitive than the ordinary measuring microscope. Extension of the principle to three-dimensional frameworks does not appear to be difficult, but has not yet been attempted.

APPLICATION TO CELLULOID MODELS

Celluloid is particularly well adapted to structural model work. It is isotropic and has a low modulus of elasticity which is sensibly constant up to fairly high stresses. It is very easy to work with either hand tools or machine tools, and it can be cemented perfectly with acetone, in which it dissolves.

Questions of similarity to prototype will not be discussed in this paper. It will only be observed that a celluloid model can be built to represent a given indeterminate structural framework with greater fidelity than a purely analytical treatment can represent it. Even when rectangular model sections are used to represent the members of a rigid frame structure, it is probable that the model will behave more nearly like the prototype than the hypothetical structure used as a basis of calculation.

PHYSICAL PROPERTIES OF CELLULOID

Samples of celluloid of different quality, age, and manufacture show marked differences in their mechanical properties. Moduli of elasticity ranging from

175 000 to 375 000 lb per sq in. will be encountered among commercial brands. Over the limits of any one sheet, however, the material is quite uniform. The following data may be considered as typical but cannot be applied quantitatively to any given celluloid. They were taken on samples of green celluloid, $\frac{3}{16}$ in. thick, known as "toothbrush stock."

For obtaining the data given herein, cantilever beams about 6 in. long and $\frac{3}{4}$ in. wide were used. The beams were tested at constant temperature in a water bath, ample time being given to allow the entire piece to come to temperature for each test.

The most striking peculiarity of the material is its creeping under load. After loading, the deflection continues to increase perceptibly for a considerable time. This fact makes it difficult to work with loaded celluloid models by conventional means. Fortunately, however, the creep-rate of celluloid is found to be independent of the magnitude of the load applied, within wide limits of stress. Stated in another way, the modulus of elasticity of celluloid is a function of time but is independent of stress up to the proportional limit. Were it not for this fact, semi-plastic materials like celluloid could not be used for accurate quantitative model studies by any known method.

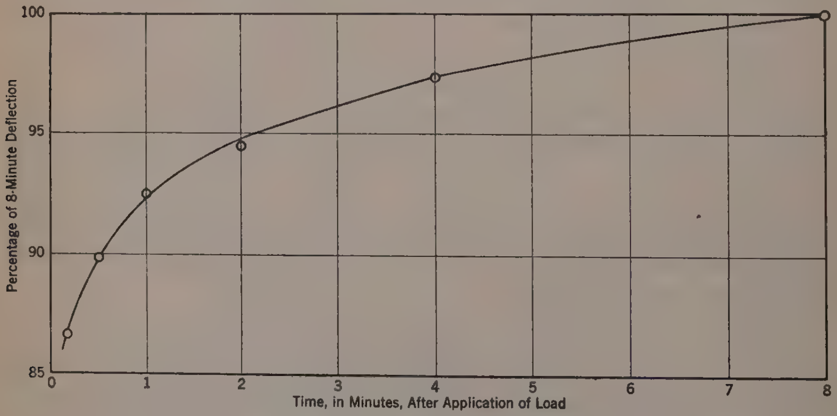


Fig. 4.—Typical Creep Curve for Celluloid (Test Temperature, — 71° F.)

A typical creep curve for stresses below the proportional limit is shown in Fig. 4. The data are plotted in terms of the percentage of the 8-min deflection reached at various times up to 8 min. Even after 8 min, the deflection had not reached a maximum. The curve was found to be the same for all stresses below the proportional limit, both for loading and for unloading. The proportional limit occurs at a stress of about 500 lb per sq in. for this material. At 1 000 lb per sq in., the secant modulus of elasticity is about 10% lower.

The modulus of elasticity is very sensitive to changes of temperature. Fig. 5 shows the behavior of the material under varying temperatures. The curve was taken for increasing temperature; after the material has been heated to somewhat more than 100° F, the curve cannot be reproduced. The sensitivity of the modulus at room temperature is nearly 1% per degree F. The coefficient of linear expansion was found to be 6.5×10^{-5} per degree F; that is, about ten times that of steel.

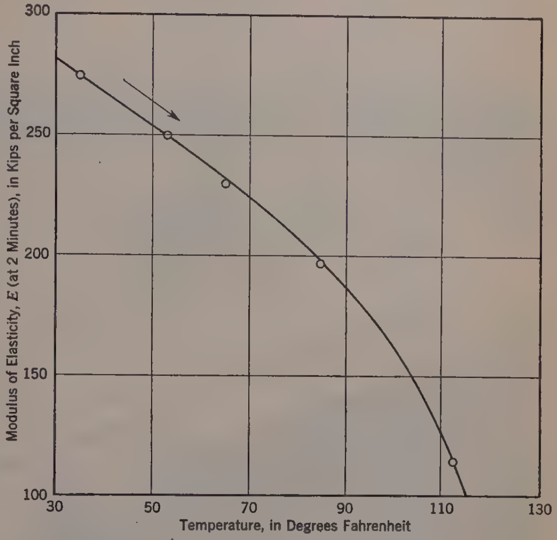


FIG. 5.—TYPICAL VARIATION OF E WITH TEMPERATURE

These data indicate the difficulties that have to be overcome in dealing with celluloid as a model material if results of high accuracy are to be obtained. Room temperature variations from day to day will often give considerable variation in modulus. In addition, temperature variations produce model movements due to direct expansion and contraction of the material. The effects of creep may be allowed for to some extent by timing readings by stopwatch, but this procedure is awkward and time-consuming. The effect of aging may sometimes be of importance.

The technique developed by the writers for overcoming the aforementioned difficulties is described herein (see heading "Test Technique with the Moment Indicator"). It was found that by using ordinary loading methods and exceptional care in timing, the average accuracy obtainable was about 10%, due to temperature and creep effects. The technique described is capable of giving results accurate to within 1%, in addition to being more convenient and more economical of time.

TEST TECHNIQUE WITH THE MOMENT INDICATOR

Since the principal difficulties with celluloid as a model material have to do with its uncertain modulus of elasticity, the writers chose a test technique that does not depend upon the knowledge of that quality. The method used consists of imposing a constant deflection at the load point, instead of imposing a constant load. A celluloid model thus loaded is free from creep, since the load applied to it follows precisely the time variation of the modulus of elasticity.

The constant-deflection method, by avoiding difficulties with creep, makes the taking of readings on the indicator an easy matter. Of course, the indicator readings taken at various points on the structure give only the relative magnitudes of the moments. The absolute values are easily determined by the application of simple statics. Thus, it is not necessary to know the value of the modulus of the celluloid; nor is it necessary to know the magnitude of the deflection imposed upon the model, both quantities disappearing from the problem entirely.

The model to be studied is fixed upon the table in such a way as to simulate the support conditions of the prototype. Fig. 6 shows the arrangement used for the Vierendeel truss discussed in this paper. A fixed hinge support is simulated at the left end; the right end is supported by the equivalent of a hinge on rollers. The model members are supported on steel balls rolling on glass plates.

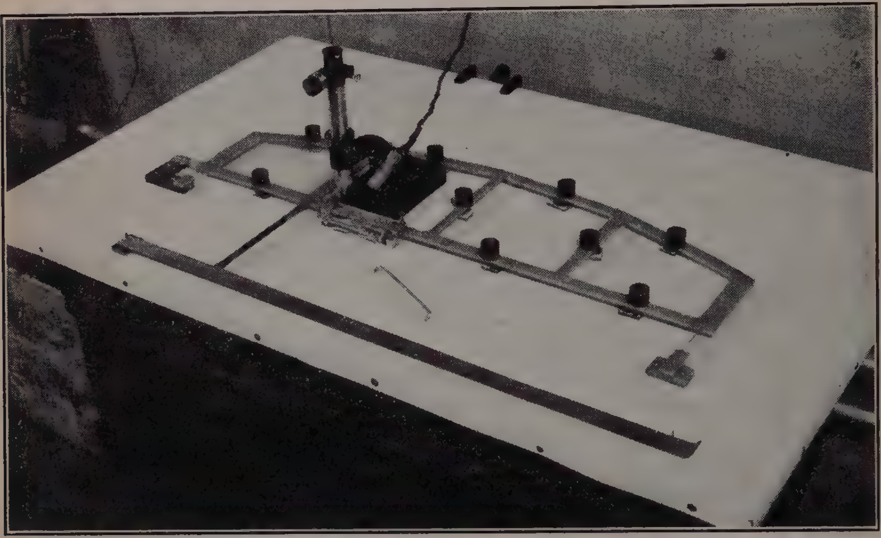


FIG. 6.—VIERENDEEL TRUSS MODEL WITH DEFLECTOR AND INDICATOR IN POSITION

The deflector is shown attached at the first panel point to the right of the left support. It is a simple stiff lever hinged at the left end, connecting to the load point by a thin steel strap. Two nails driven into the table at the free end of the deflector are used for fixing the magnitude of the deflection imposed. A counterweight, opposing the deflector, allows the deflection to be imposed first upward and then downward, thus making the net deflection twice as large as could be imposed in one direction.

In Fig. 6 the moment indicator is shown attached to a bottom chord member, and the microscope is in position for observing. The microscope is provided with a small spotlight which illuminates its field. This feature avoids the necessity of using a large lamp, the heat of which can cause annoying model movements.

It will be obvious that a number of indicator readings equal to the degree of statical indeterminancy, N, plus one is sufficient for the absolute stress determination throughout the structure. In the studies described in this paper readings were taken on all members of the structure on which it was possible to mount the indicator; that is, all except the end verticals, which were too short to accommodate the instrument being used. The extra readings were taken for the purpose of investigating the fidelity of the method.

The procedure is as follows: With the indicator fixed on any member, microscope readings of the relative index movements are made at both indices. Thus, the relative bending moments at the two ends of the gage length are determined from one instrument setting. From these two moments the moment distribution over the entire member is known at once. It was found very convenient to plot the readings (after being reduced by Equations (7)), directly on a line diagram of the truss, positive and negative moments being plotted above and below the line. The moment diagram is then constructed by drawing straight lines through the plotted points. The only unknown, E, is eliminated by applying simple statics to a section cut anywhere through the structure.

In order to secure the maximum accuracy, many more measurements were taken than would have been made in practice. Five successive measurements of the distance between index points were taken, with deflection applied alternately upward and downward. The indicator was read in two positions on each bar, with the levers first on one side and then the other. Such refinements are of value only in the investigation of the method itself, or when it is desired to work to accuracies greater than 1 per cent.

STRESSES IN VIERENDEEL TRUSS, OBSERVED AND CALCULATED

The dimensions of the Vierendeel model discussed in this paper are given in Fig. 7, and the relative moments of inertia of the members are indicated. The model was made of $\frac{3}{16}$-in., green celluloid. Butt-joints (cemented by means of acetone) were used for connecting the members, which were milled to the proper depth, since it was desired to eliminate the small errors always present in hand work. (Incidentally, it was found that when two or more pieces of the same depth were to be made, the pieces could be milled faster than they could be finished by hand, and the accuracy, of course, was much greater.)

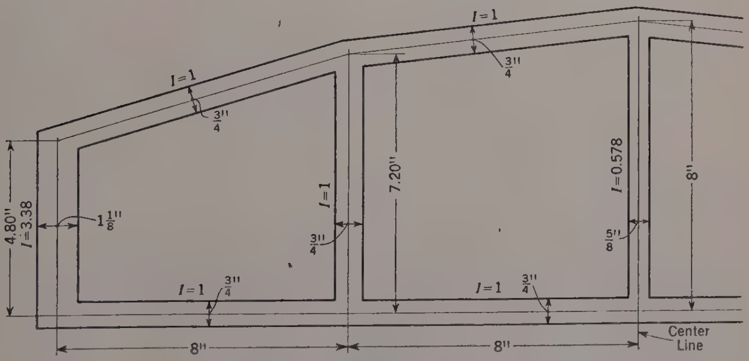
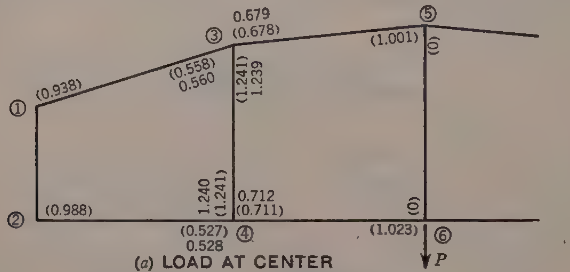
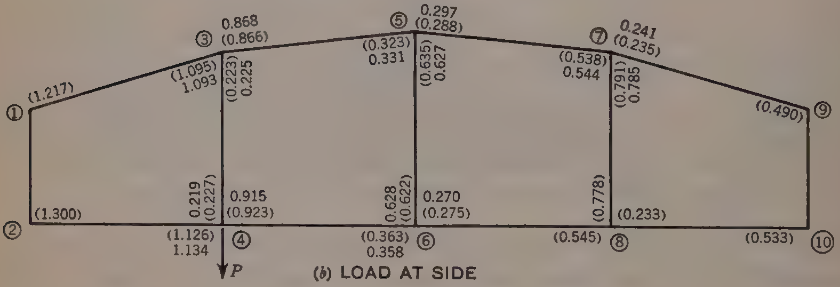


FIG. 7.—MODEL DIMENSIONS

In Figs. 8(*a*) and 8(*b*) all results are given in terms of the bending moments at the joints, in inch-pounds, due to a 1-lb load applied as indicated. The observed moments due, respectively, to a load at the center and at the side panel, are shown. The values in parentheses are the joint moments obtained by averaging all readings. The others are the joint moments as adjusted to make the summation of moments balance to zero at each joint. The joint closures are good, the maximum correction applied to produce balance at any joint being less than 1% of the largest moment in the structure.



(*a*) LOAD AT CENTER



(*b*) LOAD AT SIDE

FIG. 8.—OBSERVED AND BALANCED MOMENTS

Perhaps it should be mentioned that both the indicator and the deflector used in these tests were of home-made quality, such as can be achieved by any one who is reasonably handy with tools. Aside from the measuring microscope (reading to ± 0.0001 in.), the equipment needed for the indicator method can be made at negligible cost.

The symmetry of the model was found to be so good that only averages are given in Fig. 8, since the differences due to non-symmetry were of the same order as the differences between the individual readings. In a model made entirely by hand work, such symmetry would be highly unlikely, but for practical work, a hand-made model will be sufficiently accurate if reasonable care is exercised in finishing the members.

Upon comparing the results obtained by using only the first reading taken at any point with the result obtained by using the average of all readings taken there, it was found that the maximum difference amounted to 3.5%, with an average difference of less than 2 per cent. Thus, it may be concluded that one or, at most, two readings at each point will give sufficient accuracy for all practical work. The greatest errors due to using only the first readings are much less than the greatest errors in the theoretically calculated stresses, which are shown in Table 1.

The theoretical bending moments (see Columns (3), Table 1) were calculated with the usual assumptions for such structures. Member lengths were taken to be from center to center of joints. The effects of direct stress and shear were considered, but were found to have a negligible effect upon the results. The two sets of six simultaneous equations involved in the calculation were computed on the "simultaneous equation machine"[5] which eliminates most of the labor usually encountered in obtaining sufficiently accurate results.

TABLE 1.—COMPARISON OF MEASURED AND COMPUTED BENDING MOMENTS (IN INCH-POUNDS) DUE TO A LOAD OF ONE POUND

(a) LOAD AT THE CENTER				(b) LOAD AT THE PANEL TO THE LEFT OF THE CENTER							
Point* (1)	Measured (2)	Computed (3)	Percentage, error (4)	Point* (1)	Measured (2)	Computed (3)	Percentage, error (4)	Point* (1)	Measured (2)	Computed (3)	Percentage, error (4)
1	0.938	0.967	+3.1	1	1.217	1.247	+2.5	6- 4	0.358	0.332	−7.3
2	0.988	1.044	+5.6	2	1.300	1.286	−1.1	6- 5	0.628	0.599	−4.6
3-1	0.560	0.489	−12.7	3-1	1.093	1.088	−0.5	6- 8	0.270	0.267	−1.1
3-4	1.239	1.189	−4.0	3-4	0.225	0.194	−13.8	7- 5	0.544	0.560	+2.9
3-5	0.679	0.700	+3.1	3-5	0.868	0.894	+3.0	7- 8	0.785	0.778	−0.9
4-2	0.528	0.506	−4.2	4-2	1.134	1.114	−1.8	7- 9	0.241	0.218	−9.5
4-3	1.240	1.212	−2.3	4-3	0.219	0.200	−8.7	8- 6	0.545	0.556	+2.0
4-6	0.712	0.706	−0.8	4-6	0.915	0.914	−0.1	8- 7	0.778	0.779	+0.1
5-3	1.001	1.007	+0.6	5-3	0.331	0.326	−1.5	8-10	0.233	0.223	−4.3
6-4	1.023	1.004	−1.9	5-6	0.627	0.596	−4.9	9	0.490	0.523	+6.7
....	....	....	....	5-7	0.297	0.270	−9.1	10	0.533	0.517	−3.0

The percentage differences between calculated and measured moments are designated as "error" (Columns (4), Table 1). On the basis of the demonstrated accuracy of the moment-indicator method, these values may properly be regarded as a good measure of the errors in the theoretical results for the structure studied. The truss discussed is fairly ideal from the standpoint of theoretical analysis; it was not the intention of the writers to select a structure showing large differences between actual and calculated stresses.

FURTHER CONSIDERATIONS

One marked advantage of the method described lies in the fact that the stresses are measured by a device wholly external to the structure. In dynamical models such an approach is almost indispensable.

By arranging the instrument for photographic recording of the relative index motions, the indicator (or "moment recorder" as it is then called) has been used for studying the bending stresses in a model of a building frame vibrating as a result of earthquake motions. In problems of this nature it is clearly not practical to determine the bending stresses from records of the deflections of the model, on account of the tremendous labor involved when the model has several degrees of freedom.

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[5] "The Mechanical Solution of Simultaneous Equations," by John B. Wilbur, Assoc. M. Am. Soc. C. E., *Journal*, Franklin Inst., Vol. 222, No. 6, December, 1936.

Perhaps the most valuable application of the method may be to the study of the so-called secondary stresses in trusses. In certain types of trusses, it is well known that the conventional analytical secondary stress determination leads to serious errors, and a truly accurate analysis requires such a mass of work as to make its application practically impossible. With the sensitivity of the indicator increased several-fold, it should be ideal for measuring secondary stresses in models.

The indicator may also be found useful in certain full-sized tests. With two sticks clamped to a beam or girder, the bending moments acting could be measured accurately with less effort than by the customary strain-gage measurements. A gage length of several feet gives large deflections, the deflection increasing with the square of the gage length, as seen from Equations (2).

For routine model work, it would be desirable to have several moment indicators, in order to effect a further saving of time by avoiding moving the instrument from member to member.

In some instances, it may be desired to ascertain the absolute magnitude of the bending moment at a point in the model, without having to measure the $N + 1$ moments previously mentioned. This can be accomplished through the simple expedient of a "spring balance" made of the same material as the model, as shown diagrammatically in Fig. 9.

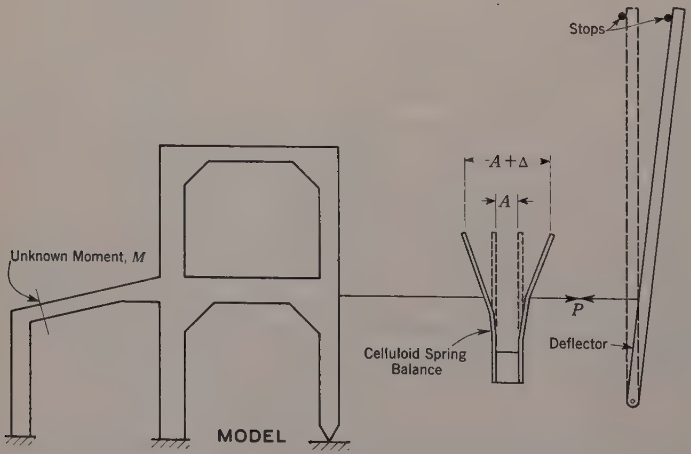

FIG. 9.—DIRECT ELIMINATION OF MODULUS OF ELASTICITY, E

The "spring balance" is made of two bars so supported as to be statically determinate. Any other calculable or calibratable system will serve the purpose equally well. The purpose of the balance is to weigh the load imposed upon the model in terms of the modulus of elasticity of the material.

Let P be the load applied due to the deflection imposed by the deflector, and let Δ be the resulting relative motion of the indices of the balance. Then, if the moment indicator reading for the unknown moment, M, is y, the following relations may be written:

$$M = K_1 E y \qquad (8a)$$

and,

$$P = K_2 E \Delta \qquad (8b)$$

in which K_1 and K_2 are known constants. Consequently,

$$M = \left(\frac{K_1}{K_2} \times \frac{y}{\Delta} \right) P \qquad (9)$$

Thus, the unknown, E, which is a function of time and temperature, is made to disappear directly.

CONCLUSIONS

The moment indicator offers a simple and inexpensive means of analyzing indeterminate structural models. The technique involved is so easy and straightforward that a minimum of skill is required. Results accurate to 2% or 3% are obtainable with an average time consumption of less than 2 min for each moment measured. By taking a few more measurements than are required, the method is made self-checking.

The writers believe that the method described in this paper is sufficiently practical to commend itself for use in engineering offices as a design tool. Any one who can read a measuring microscope can become proficient in the use of the device within an hour. The construction of celluloid models of most frames met in practice is a simple matter, requiring few tools and little skill. An office assistant under the guidance of an engineer could be entrusted with all the detail model work and observations.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

SIPHONS AS WATER-LEVEL REGULATORS

By J. C. Stevens,[1] M. Am. Soc. C. E.

Synopsis

No novelty attaches to siphons used as wasteways, but the design of siphons for close regulation of water levels, for example, in the forebay of power plants, by discharging continuously through a wide range of flow, marks a distinct advance in such structures and offers opportunities for putting siphons to uses not heretofore contemplated.

This paper describes tests made on two new siphons designed as regulators of the forebay level at the Walterville Hydro-Electric Plant, a unit of the municipal power system of Eugene, Ore. Each of these siphons is capable of discharging continuously any rate between 100% and 3% of its maximum capacity. Working in unison the pair has a maximum discharging capacity of 1 050 cu ft per sec and a minimum of 11 cu ft per sec, so that, considered as a unit, they have a discharging range of from 1% to 100% of their maximum.

The Walterville Siphons

These siphons, two in number, with capacities of 350 and 700 cu ft per sec, respectively, were put into service in the fall of 1936. They replace an old gate and flume waste from the forebay of the Walterville Plant. A part of an overflow spillway was retained by which ice and drift could by-pass the siphons. The discharge from the spillway is carried in an open flume made by extending the sides of the siphon barrel block 4 ft above its top surface. General dimensions of these siphons, in feet, are as follows:

Description	Siphon No. 1	Siphon No. 2
Length of barrel..............	127.8	123.5
Elevation of overflow crest at summit................	595.1	595.0
Elevation of crown at outlet....	550.0	550.0
Elevation of top of primer orifice	595.3	595.2

Note.—Written comments are invited for immediate publication; to ensure publication, the last discussion should be submitted **December 15, 1938.**

[1] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

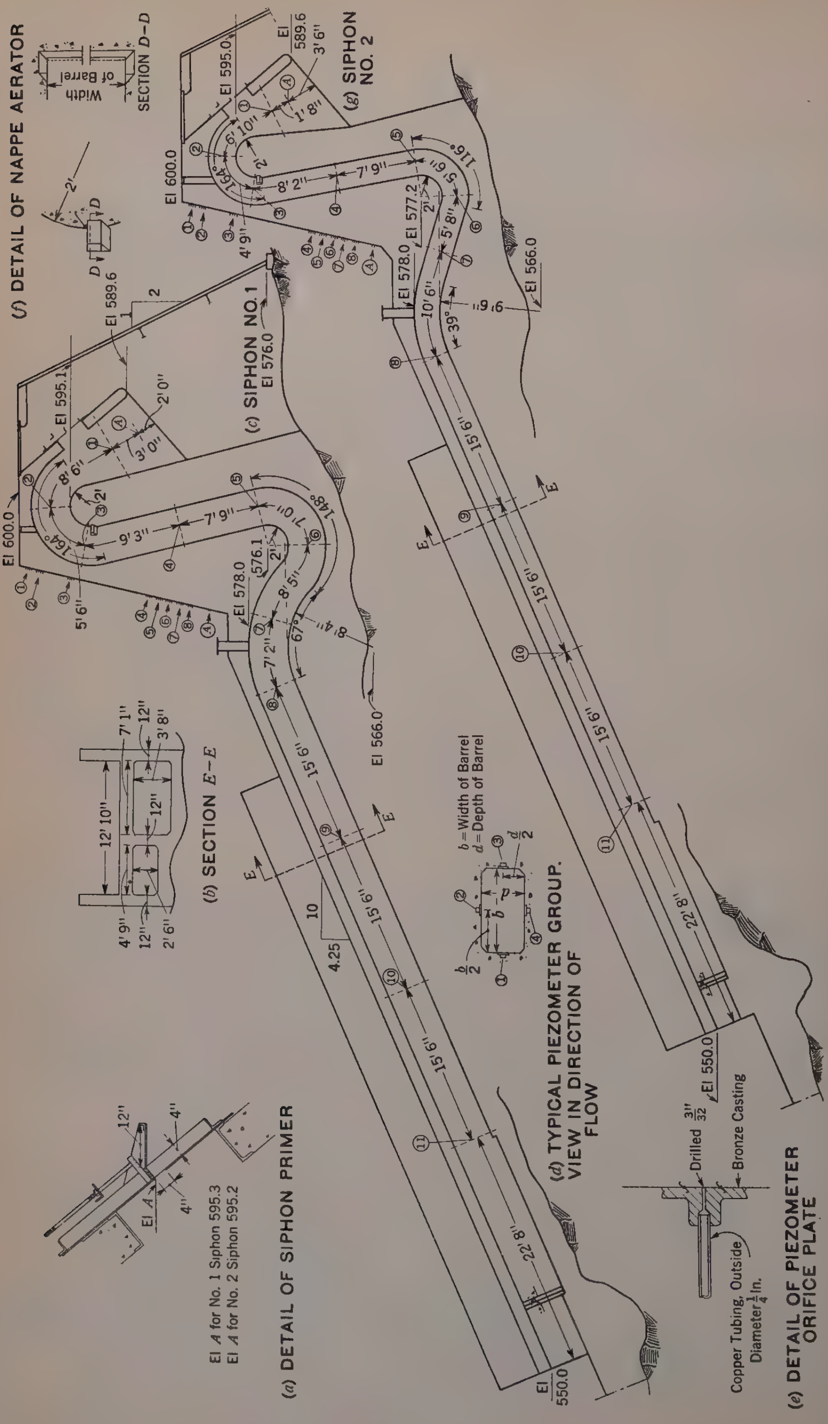


FIG. 1.—ESSENTIAL DIMENSIONS OF SIPHONS NOS. 1 AND 2, WALTERVILLE SIPHONS

Description	Siphon No. 1	Siphon No. 2
Nominal Dimensions of Barrel:		
At inlet	8.35 by 7.08	8.50 by 4.75
Section *A* (Fig. 1)	7.43 by 7.08	5.80 by 4.75
Section 1 (Fig. 1)	5.95 by 7.08	4.75 by 4.75
Section 2 (Fig. 1), to outlet	3.67 by 7.08	2.50 by 4.75

Other essential data are presented in Table 1.

TABLE 1.—WALTERVILLE SIPHONS

Section (see Fig. 1)	SIPHON NO. 1			SIPHON NO. 2		
	Distance, in feet, along the center line of barrel	Area, in square feet, at right angles to the center line	Elevation of center line, in feet	Distance, in feet, along the center line of barrel	Area, in square feet, at right angles to the center line	Elevation of center line, in feet
Inlet	0	59.0	586.7	0	40.4	586.8
A	2.00	52.5	88.5	3.50	27.5	90.0
1	5.00	42.0	91.0	5.17	22.6	91.4
2	13.50	25.90	96.9	12.00	11.84	96.2
3	19.00	25.90	93.7	16.75	11.84	93.4
4	28.25	25.90	84.6	24.92	11.84	85.4
5	36.00	25.90	77.0	32.67	11.84	77.9
6	43.00	25.90	72.2	38.17	11.84	74.0
7	51.50	25.90	75.9	43.48	11.84	75.5
8	58.67	25.90	75.4	54.34	11.84	75.9
9	74.17	25.90	69.3	69.84	11.84	69.9
10	89.67	25.90	63.2	85.34	11.84	63.9
11	105.17	25.90	57.2	100.84	11.84	57.8
Outlet	127.80	25.90	548.3	123.47	11.84	548.8

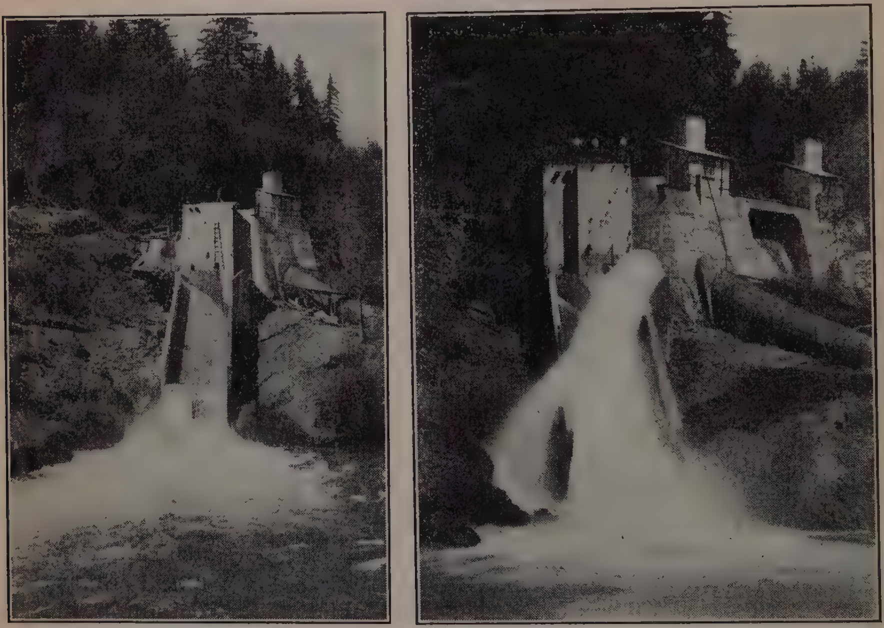
Fig. 1 gives the longitudinal profile of the siphons and essential details of the primer, nappe aerator, and piezometers. There was some question as to whether or not the long lower leg would run full. Therefore, a depression was left in the invert near the end of each barrel in which later a baffle could be built, that would deflect the water to the crown and thus keep the air exhausted from the barrel. Experience indicates that such a device is not necessary, and later these depressions were filled. As long as the siphon does not inhale air the barrel runs full to the end.

The primer is a special feature of this design. It constitutes a discharge regulator since it permits the siphon to inhale air whenever the forebay level drops below the horizontal lip of the primer. As the siphon inhales air the discharge diminishes, until the siphon discharge equals the surplus flow into the forebay. The priming orifices are adjustable both as to depth of opening and as to elevation, and each is provided with a horizontal lip above the orifice to act as a regulator of air supply, and to eliminate the drop-down curve that otherwise would appear as the water accelerates into the priming orifice.

The lower leg is effective in reducing discharge because as flow diminishes this leg runs partly full, the head on the siphon then being from the forebay level to that point in this leg where the water leaves the crown of the barrel.

The nappe aerator was provided inside the barrels just down stream of the summit crest in order to admit air behind the nappe and thus permit it to leave the face of the barrel and fall in a curtain across it, to exhaust the air more quickly. A somewhat similar aerator was included in the Leaburg Siphons.[2]

The piezometers were installed in order to determine the behavior of the siphons for use in future designs. Each piezometer has a copper tube sloping upward from the tap to a manifold on the down-stream face of the siphon block, with a cock in each tube and a connection on the manifold for a vacuum hose. The locations of these manifolds are shown in Figs. 1 and 2(*b*).



(*a*) General View (*b*) Sluice-Gate Discharging Under Full Head

Fig. 2.—Walterville Siphons

One 6-in. pipe with removable cap was provided in Siphon No. 2 and two such pipes in Siphon No. 1 in the summit sections for the purpose of venting the siphons. There is also a vent at the summit of the lower sealing curve to be used for any contingency that might arise.

At Section *A* there are two taps in each siphon in the center of the sides of the barrel. The copper tubing slopes downward from these taps to the point marked *A* on the siphon block (see Fig. 1) where they connect to a header. From this header a rubber hose extends up to a vertical glass tube laid alongside another tube connected to the forebay water. From a scale behind the glass tubes the water level in the forebay and that at Section *A* could be read simultaneously.

Siphon Discharge

Several expedients were adopted to determine the discharge through the siphons during tests. For the higher discharges two methods were used: (1)

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[2] "On the Behavior of Siphons," by J. C. Stevens, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 986.

By means of current meter measurements made in the canal about 3 miles up stream on days when the power plant was shut down; and (2), by means of a Pitot tube in the outlet end of each siphon barrel.

Flow from the forebay into the open wasteway is controlled by stop-planks, the head on which is an index of flow. This measuring device was designated the "stop-log weir." There is also a 30-in. by 30-in. sluice-gate discharging from the forebay. Knowing the forebay level and gate-opening the discharge through it was calculated as for an orifice. The stop-log weir is shown to the left of the siphon block in Fig. 2(*a*), and Fig. 2(*b*) shows the sluice-gate discharging under a full head.

On days when the current meter was used three separate measurements were made about 2 hr apart while a steady canal regimen was maintained. Water in excess of that through the siphon being tested was wasted over the stop-log weir or through the sluice-gate, or both, and was thus measured. Water was also flowing freely over the crest of the unprimed siphon which was calculated as a weir with a rounded crest-ogee type. In addition, there was a small quantity of leakage through the closed gates to the three penstocks which had to be estimated. The discharge through the siphon under test was taken as the average of the three current meter measurements made during that day, less that wasted *via* the sluice-gate, the stop-log weir, the unprimed siphon, and the penstocks. Figs. 3 and 4 show the discharges for the sluice-gate and also for the unprimed siphons. The discharge over the stop-log weir was calculated from the Bazin formula for a crest length of 6.29 ft and a height of 2 ft. A correction of 0.15 h was made in the length to allow for suppressed contraction at one end.

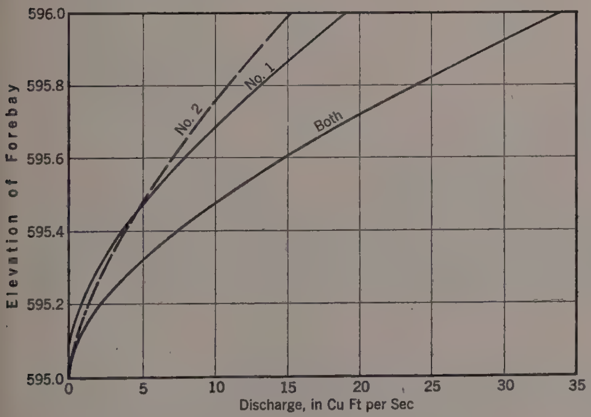


FIG. 3.—FREE DISCHARGE OVER SIPHON CRESTS

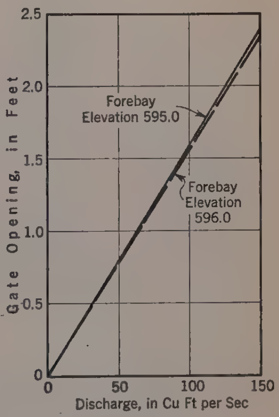


FIG. 4.—DISCHARGE THROUGH SLUICE-GATE

For the smaller discharges, current meter measurements were impracticable, and tests were made while the plant was running. Water not taken through the turbines was passed first through the siphon and then changed to the sluice-gate or stop-log weir for measurement. This manipulation involved occasional small variations in forebay level so that some slight canal and forebay storage was involved. For the low discharges that could be taken entirely by the sluice-gate this effect did not exist since the change from sluice-gate to siphon and *vice versa* could be made without affecting the forebay level.

In order to make a check on the canal measurements a frame was set up by which a Pitot tube could be held in any desired position in the outlet with the orifice perpendicular to the direction of flow. The Pitot tube measurements, of course, could be made only for siphons under full discharge with water completely filling the barrel at the outlet. These measurements are more accurate than those by the other methods since the discharge of the siphon under test could be measured directly.

There is a drop in the hydraulic grade line from the forebay level to Section *A* in which positive pressures exist for all flows. The decrease in area from the Inlet to Section *A* constitutes an effective Venturi section that was calibrated by observing this drop for each measured discharge. This drop was called the "meter-head" and was designated as H_a; the quantity, $\sqrt{H_a}$, was used as the index of flow. Table 2 gives the results of the various discharge measurements made, including all data required for plotting. A correction of + 0.01 ft was made to low heads, H_a, as it was observed that the piezometer at Section *A* stood that much above the forebay level with no flow through the siphon.

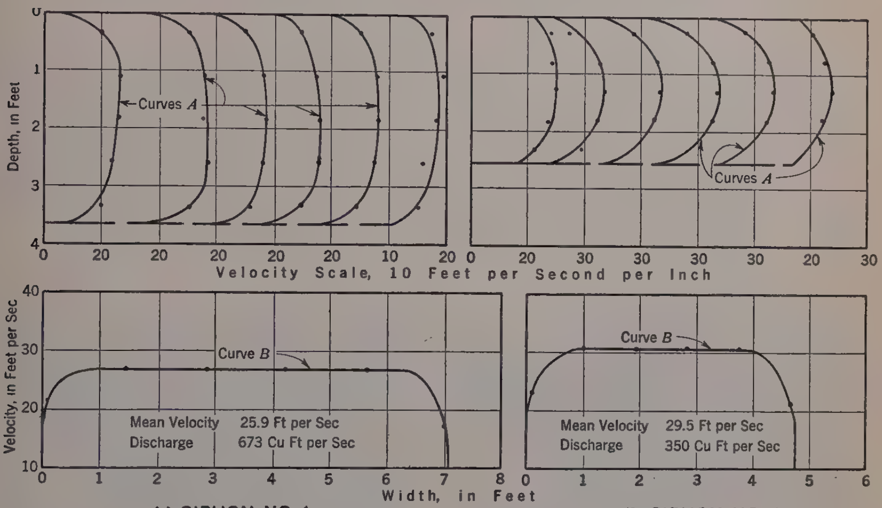
TABLE 2.—MEASUREMENTS OF DISCHARGE THROUGH SIPHONS

Date	SIPHON NO. 1					SIPHON NO. 2				
	Discharge, in cubic feet per second	Elevations, in Feet		Meter Head, in Feet		Discharge, in cubic feet per second	Elevations, in Feet		Meter Head, in Feet	
		Forebay	Section *A*	H_a	$\sqrt{H_a}$		Forebay	Section *A*	H_a	$\sqrt{H_a}$
October 14, 1936*	688	595.75	591.65	4.10	2.02	....	....	....	....	....
October 15, 1936*	594	95.58	92.35	3.23	1.80	....	....	....	....	....
October 16, 1936*	346	95.37	94.25	1.12	1.06	....	....	....	....	....
October 22, 1936†	116	95.24	95.02	0.23**	0.48	117	95.24	94.65	0.59	0.77
November 5, 1936‡	720	95.86	91.55	4.31	2.08	....	....	....	....	....
November 17, 1936†	45	95.28	95.27	0.02**	0.14	46	95.16	95.05	0.12**	0.35
March 5, 1937‡	22	95.27	95.27	0.01**	0.10	11	95.20	95.20	0.01**	0.10
April 19, 1937§	673	95.94	91.59	4.35	2.08	350	95.76	93.19	2.57	1.61

* Current meter method. † Stop-log weir method. ‡ Sluice-gate method. § Pitot tube method.
** Correction, + 0.01 ft.

For the measurements of April 19, 1937, the Pitot tube was held at five depths in six vertical sections in the mouth of each barrel. The tube itself had been rated previously, obtaining a coefficient of 0.98. It was screwed into the end of an extra strong $\frac{1}{2}$-in. pipe. A $\frac{1}{4}$-in. copper nipple was soldered into this pipe just below the upper end for vacuum hose connection to the mercury manometer. The end was closed with a cork which could be removed to "bleed" the pipe between readings.

Fig. 5(*a*) gives the results of the Pitot tube measurements for Siphon No. 1, Curve *A* showing the vertical velocity curves. The mean velocity in each vertical was determined by planimeter; these mean velocities, in turn, determine the plotted points in the horizontal velocity curve (Curve *B*), and the planimetered area of this curve divided by the width gives the mean velocity in the barrel. In similar manner Fig. 5(*b*) gives the results for Siphon No. 2. Comparison with similar curves for the Leaburg siphons[3] shows how the long lower leg has "ironed out" the effect of the up-stream curves.



(*a*) SIPHON NO. 1 (*b*) SIPHON NO. 2

FIG. 5.—RESULTS OF PITOT TUBE MEASUREMENTS

Fig. 6 shows the outlet of the barrels with Siphon No. 1 discharging 700 cu ft per sec (Fig. 6(*a*)), and Siphon No. 2 discharging 350 cu ft per sec (see Fig. 6(*b*)). The flow in the open flume is from the spillway in the forebay and that in the unfilled barrel is from the free flow over its crest. The issuing jets look white and foamy but they are really crystal clear at the exit, under the curtain of water.

Range of Discharge.—The measurements of March 5, 1937, were made using a small quantity of waste water, all of which could be measured by passing it through the sluice-gate. From the rating curve (plotted from Table 2) and from these measurements the range of discharges, in cubic feet per second, was found to be:

Description	Siphon No. 1	Siphon No. 2
Maximum discharge for minimum forebay level without inhaling air............	700	350
Minimum discharge....................	22	11
Minimum discharge expressed as a percentage of the maximum................	3.1	3.1

Priming Head.—The head at which a siphon will prime is influenced by the rapidity of the rise of the forebay. When the forebay was raised very slowly

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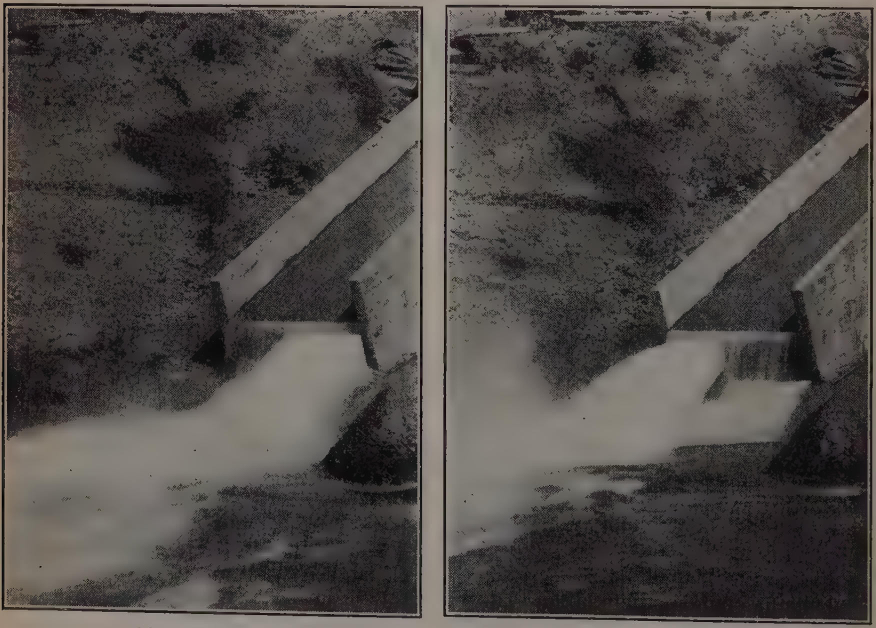
[3] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 991.

(say, 0.5 ft per hr, or less) the priming heads, in feet, were:

Description	Siphon No. 1	Siphon No. 2
Elevation of crest..................	595.10	595.00
Forebay level at time of priming......	595.40	595.23
Priming head....................	0.30	0.23

TESTS BY PIEZOMETERS

As shown in Fig. 1 there are twelve sets of piezometer taps in each siphon. In making a test it was necessary first to establish a uniform flow with a stable canal and forebay regimen. By opening each cock in turn the pressure at each piezometer was indicated by the depression of the mercury (or water) column of a **U**-tube manometer. Oscillations were virtually eliminated by a damper attached to the manometer. Since all readings, except for the forebay and for Section *A*, were negative and since the tubes contained no water and the weight of the air column could be neglected, the elevation of the manometer was of no consequence.



(*a*) SIPHON NO. 1 (*b*) SIPHON NO. 2

FIG. 6.—CHARACTERISTICS OF FLOW WITH MAXIMUM DISCHARGE

Table 3 gives the list of piezometer tests made. The discharges indicated were taken from the rating curves as plotted from Table 2.

Fig. 7(*a*) gives the essential results of the tests for Siphon No. 1 running full. The vertical lines at the bottom give the negative pressure head, in feet of water. Each bar is the average of the pressure heads at the corresponding piezometer tap from Tests Nos. 4 and 11 in Table 3. The relation of the negative pressure head to that of one atmosphere is readily seen. The pressure heads are in the order of the numbering of the taps given in Fig. 1; that is, No. 1 in the left face, No. 2 in the crown, No. 3 in the right face, and No. 4 in the invert of the barrel, while facing the direction of flow.

TABLE 3.—List of Piezometer Tests

	(a) Siphon No. 1						(b) Siphon No. 2				
Test No.*	Average Water Levels, in Feet		Meter Head, in Feet		Discharge, in cubic feet per second	Test No.*	Average Water Levels, in Feet		Meter Head, in Feet		Discharge, in cubic feet per second
	Forebay	Section A	H_a	$\sqrt{H_a}$			Forebay	Section A	H_a	$\sqrt{H_a}$	
4	595.91	591.57	4.34	2.08	700	1‡	595.72	593.20	2.52	1.59	342
5	595.30	594.34	0.96	0.98	300	2	595.80	593.22	2.58	1.61	350
6	595.24	595.01	0.23	0.48	135	3	595.22	594.55	0.67	0.82	120
9	595.28	595.27	0.01	0.10	22	10‡	595.76	593.16	2.60	1.61	350
11†	595.97	591.62	4.36	2.09	700	....	....	....	....	....	....

* Except as noted, all tests were run on March 3, 4, and 5, 1937. † April 13, 1937. ‡ October 23, 1936.

Similarly Fig. 7(b) gives the results of the tests for Siphon No. 2 running full. The negative pressure heads, in feet of water, are the average of those for each tap in Tests Nos. 1, 2, and 10. The negative pressure heads from each tap in a section were averaged to obtain the average pressure head for that section. The average heads were then subtracted from the elevation of the center line of the barrel and 33 ft added to obtain absolute pressure heads from sea-level datum. These absolute pressure heads for each section are shown as circles on the drawings.

The elevation of the center line of the barrel has been plotted against its distance from the inlet, giving a developed profile of the siphon. The numbers indicate sections at which piezometer sets were installed.

Energy Line.—The discharge divided by the area gives the nominal mean velocity at each section, the velocity head of which, added to the absolute pressure, gives the points indicated by triangles. If the barrel were filled at all points by down-stream moving water, and if the piezometers correctly indicated the true pressures, and the average of all in one set indicated the correct pressure in that section, the triangles would define a line of energy for that particular discharge. That they do not truly define such a line is due to the barrel being partly occupied by eddies, by accelerating water, by entrained air greatly expanded under the nearly complete vacuum that obtains, and the inherent inaccuracies in determining mean pressures in a section by means of the four piezometers. The absolute pressure line has been drawn to fit the points as nearly as is consistent with fundamental principles. In general, it fits the observations.

It was impossible for the energy line to conform to the triangles throughout in view of the following principles that must obtain:

(a) It can never rise above its initial elevation but must continuously slope down stream because energy is being lost throughout the entire length of the barrel;

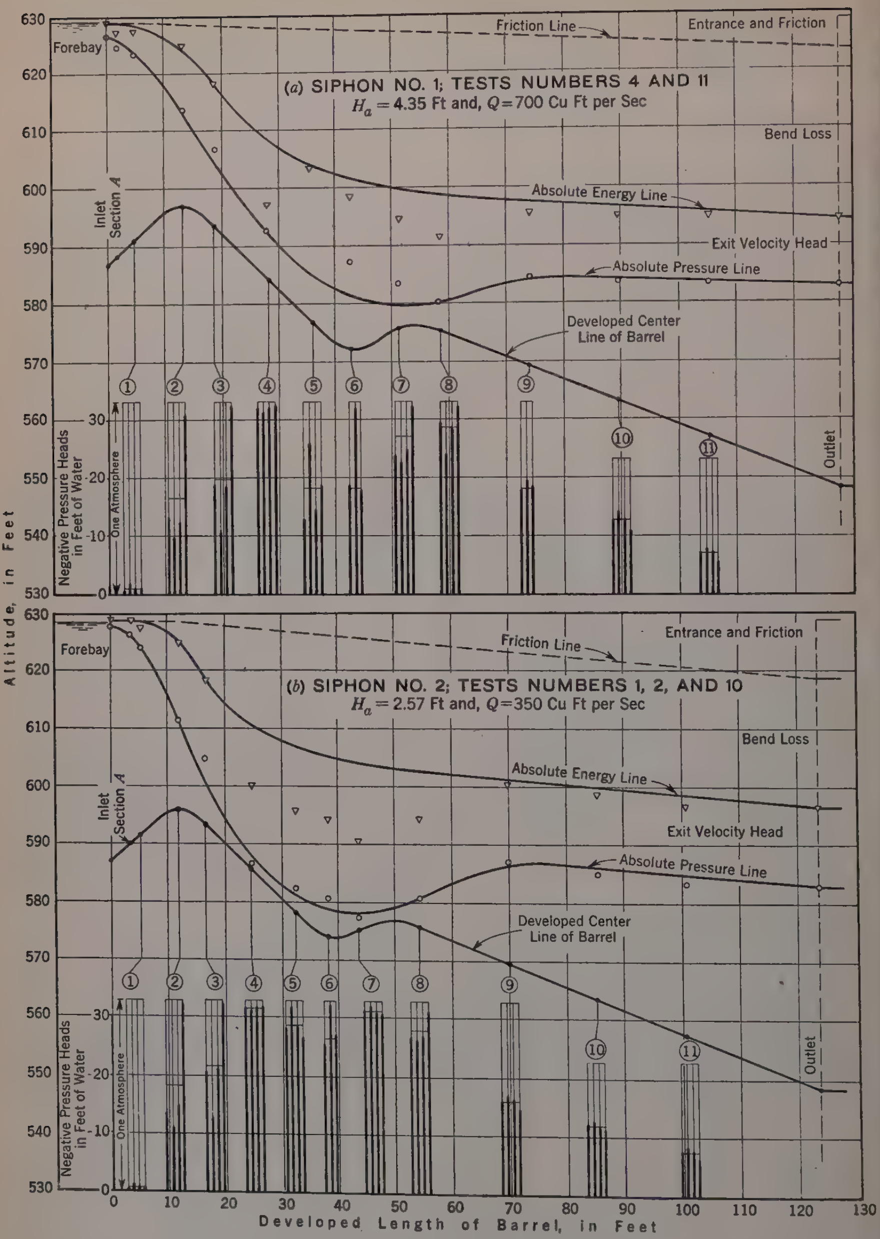


FIG. 7.—PRESSURE AND ENERGY GRADIENTS

(*b*) Its slope must never be less than that corresponding to the hydraulic friction for that discharge; and,

(*c*) Its distance from the pressure line must never be less than the velocity head corresponding to the mean velocity obtained by dividing discharge by barrel area.

Energy Losses.—The hydraulic losses are primarily of two kinds—hydraulic friction, and eddy and impact losses due to bends. The nominal friction losses were computed by the Kutter formula with $n = 0.013$, and by using the mean velocities obtained by dividing discharge of area.

The difference between the total loss and that due to entrance and friction is assigned to bends. There seems to be no doubt that the bend losses increase with the size of the barrel as there is more room in the larger barrels for eddying and spiral motion. They also are greatly affected by the distribution of velocities in the approach to the bends. Normally, the highest velocity and lowest pressure are on the inside of the bend. If the inside velocity is highest in the approach, however, the loss will be far greater than if the approach velocities are reversed.[4] Heretofore, all the experiments of flow around bends have been made with small conduits and under positive pressures. Large conduits under less than atmospheric pressures present an entirely new field of research that has scarcely been touched.

In the present case it will be noted that the difference in pressures between the concave and the convex sides of the summit bend amounted to almost 20 ft of water. (See Sections 2 and 3, Fig. 7.) Great inequality of velocities must result from the summit bend so that the approach conditions for the next bend become very complex and uncertain. To the loss in the summit bend must be added the effect of the discharge through the priming orifice joining the main stream at an angle.

In the present state of knowledge it is impossible to calculate or define the individual bend losses in these siphons. The total bend losses, however, can be determined approximately by the difference between total loss and calculated entrance and friction losses.

Table 4 gives the results of the tests for siphons running full, that is, not inhaling air. Explanation of the columns follows: Column (2) gives the average of the observed pressure heads at the four taps in each section (where the taps were plugged—Sections 6 and 9, Siphon No. 1—the average for Taps 1 and 3—at the sides—was used); Columns (3) and (5) were taken from Table 1; Column (4) is the elevation of center line of the barrel, minus (or plus) the pressure head, plus 1 atmosphere (33 ft); Column (6) is the discharge divided by the area; Column (7) is the velocity head for velocities in Column (6); Column (8) contains the absolute pressure heads plus the velocity heads; Columns (9) and (10) are the adjusted values of the absolute pressure, and energy lines, read from the curves of Fig. 7; Column (11) is the difference between adjusted values of absolute pressure and energy lines; Column (12) gives the velocities corresponding to velocity heads of Column (11); Column (13) is the discharge divided by velocities of Column (12). (This represents the portion of the barrel filled by down-stream moving water, the remainder of the area being occupied by eddies); and Column (14) is the percentage of the total area occupied by eddies.

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[4] "Flow of Water Around Bends in Pipes," by the late David L. Yarnell and the late Floyd A. Nagler, Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1018.

TABLE 4.—ENERGY LOSSES WITH SIPHONS RUNNING FULL

Section	Pressure head, in feet	ELEVATION, IN FEET		Area, in square feet	Mean velocity, in feet per second	Velocity head, in feet	Elevation, in feet, energy line of mean velocity	ADJUSTED VALUES				Useful area, in square feet	Eddy area (percentage)
		Center line of barrel	Absolute pressure line					Elevations, in Feet		Velocity head, in feet	Velocity, in feet per second		
								Pressure line	Energy line				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
(*a*) SIPHON NO. 1; AVERAGE OF TESTS NOS. 4 AND 11, MARCH 4 AND APRIL 13, 1937; DISCHARGE, 700 CUBIC FEET PER SECOND (26 CUBIC FEET PER SECOND THROUGH PRIMER); AVERAGE FOREBAY ELEVATION, 595.94; AND, AVERAGE METER HEAD 4.35 FEET													
*	....	....	628.9	....	0	....	628.9	628.9	628.9	0	....	....	....
†	+ 6.8	586.7	626.5	59.0	11.4	2.0	628.5	626.5	628.5	2.0	11.4	59.0	0
A	+ 3.1	588.5	624.6	52.5	12.8	2.5	627.1	625.7	628.2	2.5	12.8	52.5	0
1	− 0.6	591.0	623.4	42.0	16.0	4.0	627.4	624.0	628.0	4.0	16.0	42.0	0
2	−16.3	596 9	613.6	25.9	27.0	11.3	624.9	613.0	624.3	11.3	27.0	25.9	0
3	−19 8	593.7	606.9	25.9	27.0	11.3	618.2	605.2	618.2	13.0	28.9	24.2	7
4	−31.7	584.6	585.9	25.9	27.0	11.3	597.2	592.6	608.5	15.9	32.0	21.8	15
5	−18.0	577.0	592.0	25.9	27.0	11.3	603.3	585.0	604.0	19.0	35.0	20.0	22
6	−18.1	572.2	587.1	25.9	27.0	11.3	598.4	581.4	601.5	20.1	36.0	19.4	24
7	−25.9	575.9	583.2	25.9	27.0	11.3	594.5	580.0	599.8	19.8	35.7	19.6	23
8	−28.4	575.4	580.0	25.9	27.0	11.3	591.3	580.2	598.4	18.2	34.2	20.5	20
9	−18.1	569.3	584.2	25.9	27.0	11.3	595.5	583.8	597.2	13.4	29.4	23.8	8
10	−12.6	563.2	583.6	25.9	27.0	11.3	594.9	584.0	596.4	12.4	28.2	24.8	4
11	− 6.9	557.2	583.4	25.9	27.0	11.3	594.7	583.8	595.6	11.8	27.6	25.4	2
‡	+ 1.7	548.3	583.0	25.9	27.0	11.3	594.3	583.0	594.3	11.3	27.0	25.9	0
(*b*) SIPHON NO. 2; TESTS NOS. 1, 2, AND 4, OCTOBER 23, 1936, AND MARCH 3, AND APRIL 13, 1937; DISCHARGE, 350 CUBIC FEET PER SECOND (17 CUBIC FEET PER SECOND THROUGH PRIMER); AVERAGE FOREBAY LEVEL, 595.76; AND, AVERAGE METER HEAD, 2.57													
*	....	....	628.7	....	0	0	628.7	628.7	628.7	....	0	....	....
†	+ 7.6	586.8	627.4	40.4	8.25	1.1	628.5	627.4	628.5	1.1	8.25	40.4	0
A	+ 3.5	590.0	626.1	27.5	12.1	2.3	628.4	626.1	628.4	2.3	12.1	27.5	0
1	− 0.5	591.4	623.9	22.6	14.7	3.4	627.3	624.5	627.9	3.4	14.7	22.6	0
2	−18.0	596.2	611.2	11.84	29.5	13.5	624.9	611.4	624.9	13.5	29.5	11.84	0
3	−21.7	593.4	604.7	11.84	29.5	13.5	618.2	600.5	618.2	17.7	33.7	10.4	12
4	−31.8	585.4	586.6	11.84	29.5	13.5	600.1	588.0	610.4	22.4	38.0	9.21	22
5	−28.7	577.9	582.2	11.84	29.5	13.5	595.7	581.4	607.0	25.6	40.6	8.63	27
6	−26.4	574.0	580.6	11.84	29.5	13.5	594.1	578.8	605.2	26.4	41.2	8.50	28
7	−31.2	575.5	577.3	11.84	29.5	13.5	590.8	578.0	604.0	26.0	40.9	8.57	28
8	−28.0	575.9	580.9	11.84	29.5	13.5	594.4	580.6	602.6	22.0	37.6	9.32	21
9	−15.9	569.9	587.0	11.84	29.5	13.5	600.5	586.2	601.2	15.0	31.1	11.3	5
10	−11.9	563.9	585.0	11.84	29.5	13.5	598.5	586.2	599.9	13.7	29.7	11.79	0.5
11	− 7.6	557.8	583.2	11.84	29.5	13.5	596.7	585.0	598.5	13.5	29.5	11.84	0
‡	+ 1.2	548.8	583.0	11.84	29.5	13.5	596.5	583.0	596.5	13.5	29.5	11.84	0

* Forebay. † Inlet. ‡ Outlet.

The discharges through the primer orifices were determined as illustrated for Siphon No. 1 as follows:

Forebay level	595.9	
Pressure line at Section 1	590.4	
∴ Head on orifice (h), in feet		5.5
Velocity ($= 0.6 \sqrt{2 g h}$), in feet per second		11.2
Area of orifice, in square feet		2.35
Discharge, in cubic feet per second		26

The discharge through the Inlet, and Sections A and 1, therefore, was 674 cu ft per sec and, through the remainder of the barrel, 700 cu ft per sec.

Table 5 is a summary of the results for siphons when running full. It is to be noted that the total losses are greater in the larger barrel. The friction losses in the large siphon are only about one-half those of the smaller one, but the bend losses are 32% greater. The entrance losses were taken at two-tenths the velocity head at the Inlet Section.

TABLE 5.—SUMMARY OF TESTS FOR SIPHONS RUNNING FULL

Siphon No.:	Average of Tests Nos.: (1)	Head Loss Data, in Feet							Velocity, in Feet per Second				Percentage	
		Discharge, in cubic feet per second (2)	Siphon head (3)	Outlet velocity head (4)	Total energy head (5)	Entrance head (6)	Friction head (7)	Bend loss (8)	Corresponding to siphon head (9)	Actual mean outlet velocity (10)	Summit mean velocity, Section 2 (11)	Corresponding to atmospheric head (33 ft) (12)	Coefficient of discharge, outlet section (13)	Efficiency of siphon (14)
1	4 and 11	700	45.9	11.3	34.6	0.4	5.1	29.1	54.3	27.0	27.0	46.0	50	59
2	1, 2, and 10	350	45.7	13.5	32.2	0.2	9.8	22.2	54.2	29.5	29.5	46.0	54	64

The coefficient of flow (Column (13), Table 5), is the ratio of the actual mean velocity at the outlet to the velocity corresponding to the siphon head. For these siphons running full this coefficient is 50% for the larger, and 54% for the smaller, barrel; that is, they actually discharge those percentages of the theoretical discharge for these particular siphons.

The efficiency of the siphon (see Column (14), Table 5) has been defined as the ratio of the actual mean velocity at the summit section to the velocity corresponding to a head of 1 atmosphere.[5] For these siphons, Siphon No. 1 has an efficiency of 59% and Siphon No. 2, an efficiency of 64%; that is, when running full they will discharge those percentages of the theoretical maximum that any siphon whatever could discharge.

Partial Discharges.—When the forebay level is too low to keep the priming orifice completely submerged, the siphons will inhale air, the volume depending on the forebay level. As they inhale air the discharge diminishes until it just equals the surplus inflow to the forebay. The pressures that exist within the barrels for partial discharges, were also determined.

Fig. 8(a) gives the result of Test No. 5 on Siphon No. 1 while discharging 300 cu ft per sec, or 43% of its maximum. The negative heads obtained from the piezometers are shown in the same manner as in Fig. 7. The pressure line shown is the normal pressure line (not absolute pressures). It coincides with the barrel at Section 9 showing that the barrel was aerated from the outlet to that point.

In similar manner Figs. 8(b) and 8(c) show the results for Tests Nos. 6 and 9 on Siphon No. 1 for 130 and 22 cu ft per sec, respectively. In these tests the

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[5] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 1004.

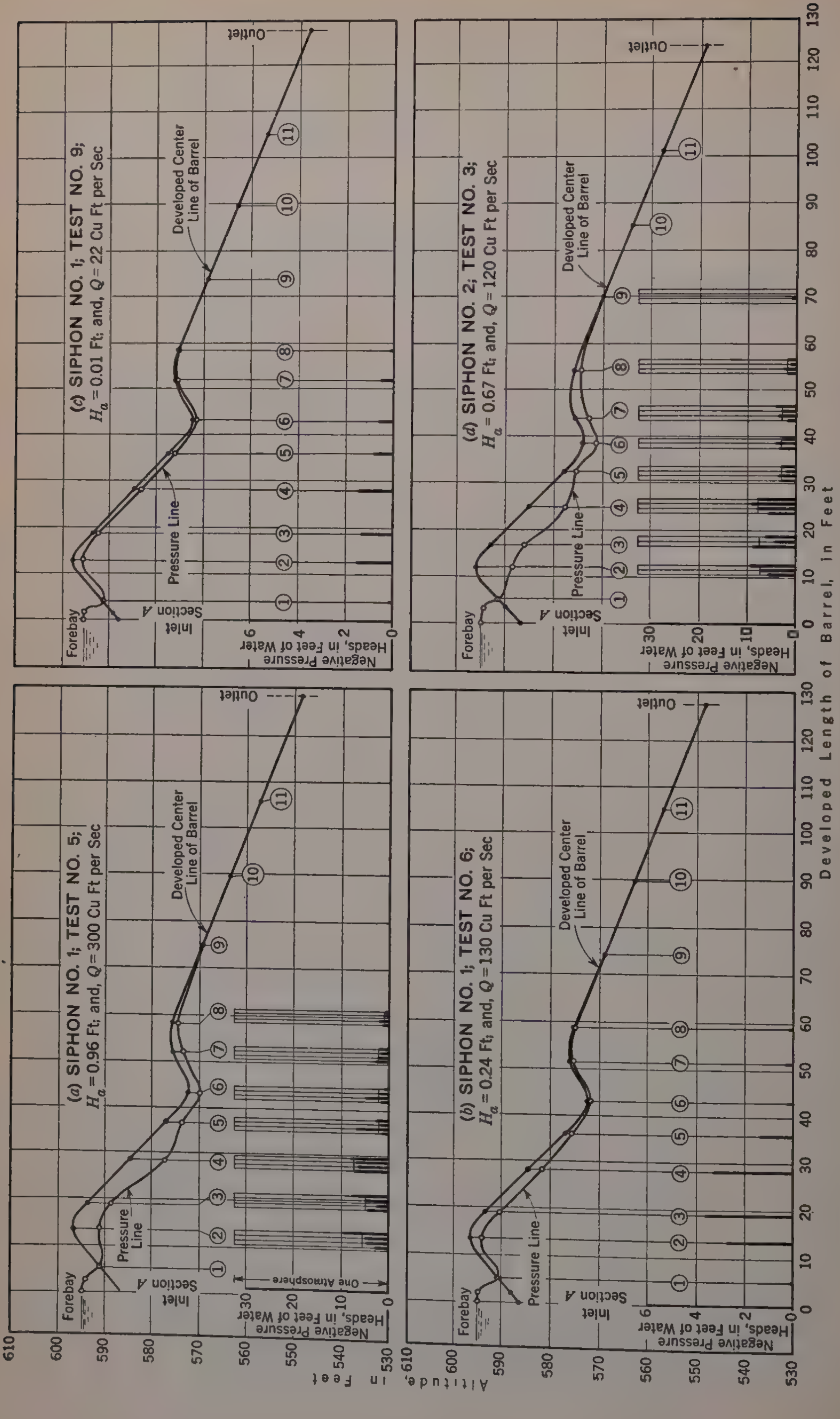

FIG. 8.—PRESSURE GRADIENTS

pressures in the piezometer manifolds, only, were observed. Negative pressures are indicated as far as Section 8 for both tests. The water flowed over the lower crest between Sections 7 and 8 without filling the barrel. This was equivalent to moving the siphon outlet to some point between Sections 7 and 8, materially reducing the siphon head below that for full flow.

Likewise, Fig. 8(*d*) shows the results of Test No. 3 on Siphon No. 2. As in the case of Siphon No. 1 the barrel was aerated from the outlet to Section 9

CONCLUSIONS

The conclusion seems justified that the larger the barrel the greater will be the eddy losses and the less the efficiency. Efficiency, however, is only a minor consideration in the design of such siphons. The most important function is sensitiveness and range of discharge, features in which the Walterville siphons excel any others known to the writer.

One important function of the lower leg is to reduce the siphon head under partial discharges. As the discharge diminishes below the lower limit, Siphon No. 1 will cease first, and if the available discharge does not become less than 11 cu ft per sec, Siphon No. 2 will take it continuously. Thus, considered as a unit, the pair has a discharge range between 1 050 and 11 cu ft per sec, or 100% to 1 per cent. Siphons with such a range of discharge for continuous operation are truly remarkable and are believed to be unique in the history of such structures.

ACKNOWLEDGMENTS

Special credit is due to the Eugene (Ore.) Water Board, of which Mr. J. W. McArthur is Superintendent, for furnishing the facilities and assistance in making these tests. Their value looms large in the field of engineering research.

The details of the preliminary tests and of the design and construction were handled by Don Umphreys, Assistant Engineer, under the direction of Hugh P. Currin, Chief Electrical Engineer. The writer acted in an advisory capacity throughout and had charge of the final tests.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

COST OF ENERGY GENERATION
SECOND SYMPOSIUM ON POWER COSTS

Discussion

By Messrs. Joel D. Justin, Frank H. Mason, W. V Burnell, Daniel W. Mead, A. G. Christie, William E. Rudolph, Louis Elliott, H. G. Gerdes, Leverett S. Lyon, John T. Madden, R. L. Thomas, Theodore B. Parker, Philip Sporn, L. N. McClellan, James C. Bonbright, R. Robinson Rowe, V. B. Libbey, and Edwin D. Dreyfus

Joel D. Justin,[32] M. Am. Soc. C. E. (by letter).[32a]—It is hoped that Mr. Whitman's paper, as well as others included in this Symposium, will help to dispel the romance which so often obscures discussions of economics in relation to power. If the public is to secure power at the minimum practicable cost, certain simple rules of economics should govern the undertaking of new projects regardless of whether private capital or Government capital is involved. Thus, if an ill-advised capital expenditure is made, the cost of power is increased, and the public pays for it either as consumer, taxpayer, or investor. Some of these simple rules of economics in relation to power supply are discussed herein.

The capacity available in any given power system should at all times exceed the existing load by a margin (reserve capacity) sufficient to insure continuity of service and, in addition to the foregoing, there should be, as nearly as may be, just enough "surplus capacity" to take care of the growth of load until the next contemplated increment of capacity can be installed and placed in operation. To maintain a proper balance between available capacity and load it is necessary to plan. "Planning" to supply future needs is not an

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Note.—This Symposium was presented at the meeting of the Power Division, New York, N. Y., January 20, 1938, and published in April, 1938, *Proceedings*. This discussion is printed in *Proceedings,* in order that the views expressed may be brought before all members for further discussion of the Symposium.

[32] Cons. Engr., Philadelphia, Pa.

[32a] Received by the Secretary May 26, 1938.

innovation of the last few years as some seem to think. The power companies and, in fact, most industrial corporations have done it for decades. Many of the public utility corporations maintain groups of engineers whose sole function is to determine as nearly as possible the future needs of the system and "plan" to meet them in the most economical manner.

When additional capacity is required its installation should be predicated on thorough economic studies by competent engineers in order to determine the capacity required, the most desirable and economic location for it, and also the kind of capacity that should be installed—that is, whether it should be steam, Diesel, or hydro-electric power. Such a study will involve not only the general design and cost estimates of proposed plants, but also the problems of transmission, distribution, and sub-stations. Sometimes, it will be found that although a huge new steam turbine in a central plant would produce the required additional power at a lower cost at the switch-board, nevertheless, when transmission and distribution are considered, it may be found that the additional annual cost will be a minimum if several smaller, less efficient, units are added at a higher cost per kilowatt in remote plants at local load centers.

Although it may be surprising to some engineers, it is believed that economically the Diesel engine may perform an important function in some large power supply systems. Thus, in the case of a long distribution line, the end of which is many miles from one of the large central stations of the system, if the load on this distribution line increases greatly, the increased demand may be met by enlarging the capacity in the central station and the distribution line. Sometimes, however, it is found that the alternative of installing a Diesel plant out near the end of the line is more economical because it is then not necessary to increase the investment in the line.

When considering whether or not a hydro plant, or a steam plant, should be installed, many factors must be given consideration if the most economic solution is to be reached. Mr. Whitman has stated some of these factors. The capital cost of a proposed hydro plant may be so great that the average cost of energy produced by it is greater than that of any steam plant in the system but, nevertheless, by reason of its peak-carrying capacity, such a hydro plant may be the most economical increment of capacity that could be installed in the given situation. One engineer referred to this problem as the hydro-paradox. In other words, it costs more, but it is cheaper, sometimes.

Some engineers, charged with the responsibility of planning development to meet future power requirements, have a leaning toward the biggest and best and may take pride in feeling that their system has the most efficient and largest units. A large unit with a minimum production cost per kilowatt-hour is not always the most economical increment of capacity for a given system. For one thing, fixed charges on unneeded capacity may more than eat up the saving in production cost. The use of extremely large units also increases the reserve capacity which it is necessary to have. A reserve requirement, which has been found suitable in many systems, is that the available reserve at any time should be either equal to the largest unit, or to 10% of the load, whichever is larger. Thus, for a 165 000-kw unit, there should be at least 165 000 kw of reserve capacity ready to go on the line in case of a break-down. For a system having a load of more than 1 000 000 kw, this may be no hardship, but for a system having a load of, say, 200 000 kw, it means that units of such great size are entirely uneconomical in spite of low production cost per kilowatt-hour.

When an additional plant is required, that increment of capacity (usually either steam or water power) should be selected which will result in the total annual cost of power supply for the entire system being lower over a term of years than it would be if any alternative increment of capacity were added.

Throughout most of the United States the development of power systems on an economic basis has resulted in the inter-connected combined power systems which Mr. Whitman has discussed. The power supply of these systems is derived from both steam and hydro-electric plants.

Even some engineers are prone to discuss the thesis: "Is steam power cheaper than water power?" Such questions should be relegated to the high school debating societies because no general and truthful answer is possible. In sections where topographical, water supply, and load conditions are favorable, it is usually economic to supply a part of the requirements of the increasing load by the construction of hydro plants. The peak-load carrying advantages of water power are so great that it appears improbable that any near-by development will crowd the hydro plant out of the economic picture. Thus, in 1922, 31.0% of the total capacity of privately owned public utility plants was hydro capacity. In 1937, the corresponding value was 27.5 per cent. The latter may have been adversely affected by the extensive construction of water power plants by the Federal Government.

Power supply investigations for various systems have convinced the writer that, for most territories, there is a fairly definite economic hydro ratio which may be defined as that ratio of hydro capacity to annual peak load that will give a minimum total annual cost of power supply for the given system.[33] This economic hydro ratio may range from 100% (as in some sections of Canada where fuel is very expensive and suitable hydro sites with abundant water supply are common) to a very small percentage in territories where fuel is very cheap and where favorable water-power sites are almost nonexistent. The interesting fact is, that for considerable territories covered by inter-connected systems, the economic hydro ratio appears to be quite constant for that section.

Mr. Whitman makes the statement in connection with a Government hydro project (see heading, "Federal Hydro-Projects") that "it was testified that the Government would pay only 3% for its money, therefore, the rate of return required would be only this rate; whereas, private companies had been compelled to pay about 7% for their money." When all factors are considered, 3% Government money is in reality somewhat more expensive than the 7% private money required by privately owned utilities for capital expenditures.

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[33] "Power Supply Economics," by Joel B. Justin and William G. Mervine, John Wiley & Sons, Inc., New York, 1934, p. 142.

In order to determine the true cost of money for a proposed Government project some such set-up as the following should be made. (It is herein assumed that Government construction and operation are just as efficient as if the project was privately owned and that in each case the proposed project would be loaded to the same extent):

Cost of Money to the Government for a Power Project Amounting to $1 000 000 (The Term, "Government," as Used Herein Means All Government—Federal, State, and Local):

(1) Interest on Government investment in power project:

3% on $1 000 000.......................$30 000

(2) Taxes on privately owned public utility which Government will have to forego if it builds and operates the project itself; 15% of revenue (in many cases this is more than 16%) which, on a 1 to 6 ratio would be:

15% of $166 700......................... 25 000

(3) Individual Federal income taxes of investors on dividends and bond interest of alternative privately owned project which Government would have to forego if it builds and operates the project itself. Since such investors are ordinarily in the higher than average income groups, this will probably average 15% of the 7% return received by them, or,

1.05% of $1 000 000..................... 10 500

(4) State and local taxes on the income of investors which Government will have to forego if it builds and operates the project itself. In Pennsylvania, this takes the form of a personal property tax, 0.4% State plus 0.4% county on values, or,

0.8% on $1 000 000...................... 8 000

Total annual cost of money to Government for $1 000 000 power project...............................$73 500

Hence, on this basis, the true cost of money for the Government project is 7.35% as compared to 7% for the privately owned project.

Item (3) is the writer's estimate as he does not know of any statistics on the subject. In the case of any individual investor, this item would be the highest increment of the income tax which he pays. There is also the question of multiple taxation in some cases which would be very difficult to evaluate. Item (4) is precise for the Commonwealth of Pennsylvania, except that it does not apply to the securities of corporations incorporated in Pennsylvania, but may vary greatly elsewhere.

Although it is believed that the foregoing tabulation approximates the truth in many cases, it is presented merely to illustrate one of the set-ups which the Government authorities should make as the result of extensive study to determine whether or not it is advisable to construct a Government project. This statement assumes, of course, that the only reason for the Government to build a power project would be to obtain power for the people at the lowest practicable total annual cost.

FRANK H. MASON,[34] M. AM. SOC. C. E. (by letter).[34a]—Each author of this Symposium has performed a service in helping to give a broader view of the subject. Professor Barrows covers the field of hydro-generated energy with the exception of pumped storage plants, which are not common in the United States. This paper is valuable in revealing such variations in the cost and the output of different types of plants as the following: The higher head plants normally cost less; the capacity and capacity factor have a great influence on the cost of the plant and the cost of power; and, the increase in size of the plant generally lowers the cost of operation and maintenance in terms of the capacity or the output.

Professor Barrows also gives the reason for, and value of, over-capacity of a plant to some systems, and the advantage of using water power for peak capacity in a combined hydro-electric and steam system. He gives a summary of what other engineers have done and think, as a comparison and aid. Even a casual reading of his paper shows that the variations in cost of a hydro plant and the output are dependent on the locality, the type of plant, and the object of its construction.

Too many people in the past have sought, or quoted, an average price for construction and for generation without knowledge of the requirements of the service, or the location of the plant. The Government proposed to have a yardstick for the measurement of cost. The examples, either because of their large size, the subsidized cost, or the lack of information on the cost, have failed to furnish material for a fair comparison.

The fifteen years since 1923 have brought about a remarkable advance in the type and knowledge of equipment for hydro-generation and power development construction. Who has brought this about and paid for the cost of present-day advancement? Certainly no one but the private company, the manufacturer, and the private laboratory. If the private company could rebuild its plant with present, up-to-date equipment at subsidized values and with low interest rate, and then operate without much, if any, taxation, a low cost would be obtained; but, after all, the real costs depend on the sale of output and not on a plant's capacity to generate power. Some of these Federal plants for years to come might just as well be back in the woods of Canada because the plants will not have a sale for power. Consequently, the actual output costs will rise, or there will be years of fixed charges to be met.

It has taken years of time and much expense to acquire loads, and to perfect equipment. Even now, the increase in demand for a better and surer type of service, the unknown question of obsolescence, and especially the increased taxation, leave the future unknown. It is certainly a proper time for a complete review of the subject of power costs.

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[34] Chf. Engr., New England Public Service Co., Augusta, Me.

[34a] Received by the Secretary May 31, 1938.

W. V. BURNELL,[35] ESQ. (by letter).[35a]—In his paper, Mr. Scharff has made a distinct contribution to the troublesome subject of depreciation. He has lifted the veil of mystery that has long enshrouded the subject by demonstrating that many facts, which hitherto have been determined largely by judgment, are, after all, susceptible of reasonably accurate and reliable determination. If the business of the electric industry is to be governed hereafter in accordance with the facts concerning existing depreciation, much is to be gained from a careful consideration of Mr. Scharff's proposals.

In any forward view of the matter, however, it is incumbent upon the management of electric utilities to inquire also into the regulatory aspects of the problem. No matter how sound and well supported a professional determination of the facts may be, it must first be accepted by the regulatory body and the Courts, before it can safely be adopted by a regulated utility as a business policy.

As to the attitude of these regulatory bodies, if one may judge from the revisions that have been made recently and which are currently being made in the accounting systems which they prescribe, it is leaning definitely in the direction of a radical departure from previous practice. There is ample reason for the conclusion that the commissions, generally, are now in favor of a depreciation policy which is essentially actuarial in nature and based on some application of the principle of life expectancy. Along with this trend of thought, there goes the parallel conviction that the depreciation policy which is to be adopted for the purpose of utility accounting should correspond with the basis adopted for the determination of permissible income.

Depreciation is defined as loss in value. The purpose of depreciation accounting is to provide a means by which the losses may be recouped by the investor. In principle, it matters little how the recoupment is provided for, just as long as the investor receives the income to which he is entitled during the life of the investment and the principal when the income terminates.

Until recently the electric industry, in accordance with the privilege extended to it by most of the regulatory commissions, has seen fit in most cases to follow the retirement reserve policy. To a considerable extent this policy has relieved the present customer of depreciation charges to compensate for losses in value which, although anticipated as a future probability, have not yet occurred. Under this policy, some of the losses brought about by improvements in the art have been assessed against the prospective beneficiaries of those improvements. The tendency now seems to be to require the present customers to accept a much greater share of the burden.

In this apparent transition from one depreciation policy to another, the industry is confronted at once with two important considerations. In the first place, there is a disposition to make a retroactive application of the new rule of the game and, by this process, to assess against the utility a loss in value for which the present investor would have been compensated if the new rule had been in effect hitherto. This is not "cricket" by any means, but the industry nevertheless is confronted with the danger. In the second place, the newly adopted rule implies the necessity of increased annual appropriations to the depreciation reserve and a relatively higher schedule of rates to provide for them.

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[35] Appraisal Mgr., Stone & Webster Eng. Corp., New York, N. Y.

[35a] Received by the Secretary June 3, 1938.

As to the question of retroactive application of a new rule which deprives the industry unfairly of some part of its property value, there seems to be no available alternative except for the utilities to insist upon their rights. As to the policy to be followed in the future, however, the industry has been "placed on notice" that the old rule may no longer be honored. It may be that a contention can be made successfully against the new rule, but all will agree that at least there is considerable doubt about it.

It may be stated also that the industry is confronted with a number of new competitive aspects of the situation which are purely economic in character. Aside from any such economic aspects, however, and in the light of the regulatory trend alone, would it not be the part of wisdom for the electric utility companies to "drop an anchor to windward" by a careful analysis of their operating charges for depreciation, and also as to the sufficiency of their charges to the customers for service to provide for the higher cost of power which may result from the new regulations?

DANIEL W. MEAD,[36] PAST-PRESIDENT AND HON. M. AM. SOC. C. E. (by letter).[36a]—In 1924 the writer was drafted by the Power Division of the Society to prepare a paper on "The Economics of Hydro-Electric Development"[37] for the Spring Meeting on April 9, at Atlanta, Ga. For simplicity the analysis was confined to single independent hydro-electric developments and should perhaps be somewhat modified when considering developments in connection with extensions of properties already in operation and with combined projects having two or more objectives. It is obvious that, when considering new developments made in connection with extensions of power properties already in operation, conditions are somewhat changed. For example, the long and expensive process of developing a market may be entirely eliminated because the power of the new plant, upon completion, may be turned at once into an operating distribution system. In combined properties having two or more objectives, the distribution of first costs and operating costs may be very complex and indeterminate. Appendix A of the writer's paper[38] includes a simplified analysis of "Factors of Economic Expediency"; Appendix B, "Capitol Costs"; and Appendix C, "Annual Expenses."

The writer has re-examined these analyses and finds no additions which he would make at this time and nothing which he would eliminate. Similar outlines for plants which develop power from heat sources can readily be made. The writer believes that very little difference in opinion is possible among power engineers as to what should, in the general case, be included in such an analysis.

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[36] Prof. Emeritus, Hydr. and San. Eng., Univ. of Wisconsin; Cons. Engr., Madison, Wis.

[36a] Received by the Secretary June 6, 1938.

[37] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 158.

[38] *Loc. cit.*, p. 186.

In view of the discussion now before the public as to the actual and relative costs of hydro-electric and steam-electric power, and in view of the pretense of the Federal Administration to establish so-called "yardsticks" to measure the legitimate cost of power as generated by public utilities, it seems fortunate that the subject of power cost is now open to discussion in the form of the present Symposium.

To those familiar with power-cost analysis the subject cannot be regarded as greatly complicated; but to those unable to make such an analysis, the subject is complex on account of the numerous factors which enter into its consideration, all of which must be properly evaluated in order to determine the proper answer to any particular problem.

No simple rule, measure, or criterion can be applied under the great variations in circumstances that occur when the entire United States is considered, and often wide variations of conditions occur within quite narrow geographical limits. This is particularly true when comparing public and private power plants as to first cost and cost of operation.

Mr. Page indicates some of the differences in cost between power plants constructed by public and private agencies such as "taxes," "franchises," and "State and Federal income taxes." The matters of "interest on first cost" of the project and of "interest during construction" are also of great importance and are seldom considered in Federal projects. Even if they are considered, the considerable difference in interest rates (often urged as a condition favorable to public ownership) would make plant and power costs not fairly comparative.

It is unfortunate that the public, including both State and Federal legislators, seems to regard this matter of power costs as comparatively simple and easily determined because, in fact, it is a problem for the trained power analyst. The idea of the average legislator on this subject is illustrated by the following example: About 1910 the State Legislature of Wisconsin passed a water-power law by which an attempt was made to confiscate the privately built water powers of Wisconsin on the ground that they were natural resources which legally belonged to the State. Prior to the passage of this law, extended hearings were held by a joint committee of the House and Senate of the State and the writer spent considerable time in trying to explain to this committee the basis of hydro-electric and steam-plant power costs. Among the questions asked the writer, was: "How much does it cost to develop a horse-power from water?" The writer explained that such cost would vary largely with local conditions, size of plant, nature of equipment, and the point at which the resulting power is to be measured. For example, the cost of installation might vary from $35 per hp, measured at the turbine shaft of a high-head impulse wheel in the Far West to $350 per hp, for a low-head plant in the East with power measured at the switch-board of the customer, say, 100 miles from the power-house; and that similar differences in steam-power cost depended on the size and character of equipment and the great variation in cost of fuel and labor at different points in the country. The writer thinks that it was the opinion of at least the so-called "progressive" members of the Committee that he was endeavoring to obscure the issue rather than to clarify it; when, in fact, he was doing his utmost to present this somewhat complex subject fairly and in the simplest language possible. He explained that the question is similar to such questions as, "What does it cost to build a house?" and can be answered only with specific conditions in mind.

Assuming that the writer's estimate of hydro-electric costs is fairly accurate, at least for the purpose of illustration, it is quite evident that the $35 per hp cost of power development at the turbine shaft in the Far West, high head plant is not an honest nor a far "criterion" or "yardstick" for the cost of power development from a low-head plant in the Central or Eastern States where the actual cost of development was $350 per hp, delivered at the switch-board of the customer 100 miles distant. Neither is the reverse true.

In discussing "Public Ownership of Power" and the so-called "yardstick," Arthur E. Morgan, M. Am. Soc. C. E., states[39] that "it is imperative that records and accounts be honest, and fair and open, and that there be no hidden elements of subsidy. The very fundamental element of such comparison is honesty, fairness, and openness in measurement." He might have added that the circumstance of development of all plants compared to this "yardstick" should be essentially similar or identical.

It is obvious, of course, that all plants competing for business in any given community must furnish power at a competitive price regardless of their cost of construction or of the cost of delivering power. If they cannot compete they must go out of business. There cannot be any real competition between Federal and privately owned plants, however, as the latter are regulated and controlled by the Government and can be eliminated at any time by Government fiat in which the factors that of necessity control private investments may receive little consideration. The United States attorneys on the case between the TVA and the private utilities which was heard at Chattanooga, Tenn., in 1937, stated that, if Congress so directs, the Federal Government can furnish power for nothing. This is doubtless true, but in so doing the Administration can scarcely expect engineers or even the public to accept such prices as a fair and honest "yardstick" for the measurement of public utility charges.

The so-called "yardstick" is purely fantastic and actually meaningless except as a political catch word or to the man who assumes that the problems of the cost of power plants and of power generation are simple, and of such uniform and constant character under all circumstances that a criterion can easily be established that is widely applicable. This idea is entirely erroneous. All power engineers know that, in general, the only way to determine with any degree of accuracy the fair cost of a power plant or of power generation is to determine all the circumstances under which the power is generated, delivered, and used.

The determination of the cost of a power plant and the fair price for the power which it furnishes, by building a competing plant under entirely different circumstances and operating it together with many other objectives whereby both the first cost and the cost of operation will be hopelessly obscured, is an idea which would never occur to an engineer. Such a method of determining power costs is perhaps fairly comparative to the way in which most political questions are now solved.

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[39] *Atlantic Monthly,* September, 1937, p. 342.

While the determination of power costs is complicated to the layman, the principles involved are so simple that in the main the writer would expect little difference among qualified engineers as to those matters which must enter into any fair consideration of the costs from any single plant or combination of plants. Differences in opinion concerning power cost arise, however, not in the theory but in the application of principles of concrete cases.

When a hydro-electric power development is only one of a number of objectives undertaken in a common development and it becomes necessary to separate the costs of power development from other objectives, real or imaginary, that are included in the combined project, there probably always will be difference of opinion, even among engineers, as to the proper proportion of costs to be distributed to the various objectives. Fair and just proportion of charge, which should be assigned to any one objective, cannot always be fully demonstrated and will always result in a wide difference in opinion.

For example, in the Grand Coulee project on the Columbia River, in Washington, the objectives are said to be navigation, flood protection, irrigation, and power. In the Tennessee Valley the objectives are said to be navigation, flood protection, power, reforestation, erosion prevention, the manufacture of fertilizer, and national defense. In each case power is assumed to be incidental to more important objectives that are legitimate undertakings of the Federal Government, which cannot undertake power development as a main or principal objective. It seems quite evident that in all cases where more than one objective is declared in the enabling act, the distribution of fair and just costs to each objective is difficult, if not impossible, and must to a large extent, be, and remain, uncertain. For this reason the attempt to establish, as a criterion or "yardstick" for measuring either the cost of development or the cost of power, any power plant having an uncertain cost of construction and an equally uncertain cost of operation is fallacious. Such a criterion is of no value in measuring the true or fair cost of development of the cost of power for plants built under quite different circumstances.

It is evident that there may be various points of view in particular cases. If, for example, the present Tennessee Valley improvements, in their present form, were needed and warranted for flood-protection and navigation purposes, and after construction and completion, power plants were added, the fair cost of development might include only power-house equipment, transmission, etc. Such cost certainly should include interest, engineering, supervision, freight and mail subsidies, etc. Even then it could hardly be considered a fair "yardstick" to measure the cost of power to an independent development where power only is the main objective and many other factors are involved.

In the Tennessee River case the high dams are probably an actual detriment to navigation when compared with low dams built purely for navigation purposes, and have only a limited value for flood protection. As a matter of fact, the height of dams was undoubtedly determined from the power standpoint. Suitable navigation and all flood protection warranted by the condition on the Tennessee River could have been furnished at a fraction of the cost which has apparently been charged to these objectives; and the objective of water power apparently has been charged with less than one-third the costs which the writer believes should be fairly attributed to it.

Neither the Federal Administration, Congress, the State legislatures, nor any other authority, corporation, or individual, can in any way establish a fair and just measurement of what power will cost, or the fair and equitable price at which it should be reasonably sold, except by considering the individual circumstances of construction and operation.

In other words, the so-called "yardstick" for the cost of power is a purely fantastic and impossible ideal, and when attempted is only fairly applicable to the plant or combination of plants created for such an unnecessary purpose. Such a term sounds well to the layman and, therefore, is useful to the politician to mislead the people and to secure popular approval; but it is practically meaningless to the engineer except perhaps in the case of plants when the conditions of duplication may be so uniform that almost identical results can be expected to obtain.

False estimates of the cost of power for either Federal, State, or private power plants will only deceive the engineer where costs are unknown or where "hidden elements of subsidy" cannot be determined. A clear and honest statement of facts and conditions or an investigation of these conditions, if permitted, will demonstrate to any qualified engineer or board of engineers, the fair cost of a power plant or power plants and the fair cost of power which such plants can or do deliver to their customers.

On the other hand, by unfair and dishonest statements and inaccurate or erroneous records of conditions and costs, the public may be deceived—"all of the public part of the time and part of the public all of the time."

The writer agrees with Mr. Page that it is time for engineers "to assist, honestly and whole-heartedly, in seeing to it that the public weal is placed foremost among the objectives of the power industry." He also believes that there is no less of a duty due from the administration and its officials to deal frankly, honestly, and intelligently with the public.

Mr. Page's paper is a frank and fair statement of the factors that enter into the problem of power costs, and the writer finds little to which he desires to take exception. He is unable to agree, however, with the apparent assumption that many of the "large Federal multiple-purpose projects" are warranted at this time, even on the basis of doing something possibly to aid in recovery from the depression. To add to the burden of the taxpayers of the nation the enormous sums made necessary by many of these entirely unwarranted projects can only prolong the depression and make recovery more difficult. The lack of information concerning these Federal Government multiple-objective projects and district power plants leads to the conclusion that they have not been duly considered but have been hastily adopted as projects which would furnish opportunities for labor and might possibly aid in recovery from the depression. It is evident that in most, if not all, cases there has been no proper and detailed consideration of their plans, cost, desirability, necessity, or soundness prior to their adoption and authorization. Under such conditions it must be expected that many ill-conceived and unwarranted projects have been included and that great waste will be entailed.

Undoubtedly, much misinformation has been issued on the subject of power costs—some of it propaganda, to mislead the public for or against either private or public ownership and operation of power plants. Some of the information on this subject is furnished without sufficient explanation to limit its use in a proper manner.

A committee of the U. S. Chamber of Commerce in 1936 published a table[39] entitled "Federal Power Projects" which gives: (1) The name of the project; (2) the approximate expenditures to June 30, 1935; (3) the ultimate cost (estimated); and (4) the ultimate horse-power (estimated). Any one familiar with power matters should know that all these projects are multi-objective in character and a reading of the report so indicates; yet in a paper read at the Sixth Midwest Power Conference held in Chicago, Ill., on April 22, 1936, this table was taken, unmodified, as a basis for estimating the costs of Federal hydro-electric power plants and power costs. This unwarranted use of the data destroys almost the entire value of an otherwise excellent paper and leads to suspicion of intended propaganda adverse to Federal hydro-electric power projects. The paper is so denounced in a "letter of submittal" and in a report sent to the President of the United States by the Chairman of the New York Power Authority[40] in which the public development of hydro-electric power projects is presented in an equally fair and logical way.

Every power engineer knows that the profession is entirely ignorant of the probable cost of those Federal Power Projects, much less as to what costs will ultimately be charged to them by the Federal Power Authorities. It is doubtful if the public or even the Engineering Profession will ever know what fair cost should be attributed to these power installations or power costs in these Federal multi-objective installations where many of the objectives are largely imaginary and often introduced only to assure the legality of Government development.

The writer believes that the question of hydro *versus* steam power plants can never be settled by such general one-sided and unfair presentation based on unknown or imaginative data. All power engineers know that each problem is a question of local conditions and must be studied and solved as such. After all, the power market or the probable demand for power should be the true reason for any installation and this important matter is ignored in many of these discussions as though power were of value, whether used or not.

Even if the first cost of the development of these projects could be established, it is certain that quite a different result would be found if the delays and consequent cost of market development were considered. Many a private development has failed of success due to long delays in the establishment of a profitable market. In Federal projects such expenses seem never to be considered, as they can be met at the expense of the taxpayer, who will probably never appreciate how much he is called upon to pay for the development of unwarranted and ill-advised Government projects, not only for power but also for navigation, irrigation, flood protection, and recovery.

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[39] Report of Committee on Water Resource Policies, March, 1936, p. 22.

[40] H. R. Doc. No. 52, 75th Cong., 2d Session.

It is certainly true that honesty, intelligence, frankness, and fair dealing should be introduced into all public and private relations; for to the present time neither Government authorities nor business interests have a monopoly on either honesty and frankness or misrepresentation and propaganda.

A. G. CHRISTIE,[41] ESQ. (by letter).[41a]—The elements entering into the cost of heat-generated energy have been clearly described in the paper by Messrs. Hirshfeld and Van Duzer. Their cost estimates represent a painstaking analysis of available data and a careful selection of average values for inclusion in the curves. Messrs. Hirshfeld and Van Duzer leave little for discussion other than to amplify some of the factors they have considered. They emphasize the fact that any costs presented in the paper are not representative of the industry as a whole but are only the logical deductions that one derives from the data assumed in each case. They stress the fact that conditions of location, load, and fuel vary so widely that costs under the several conditions cannot be equal; nor are costs truly comparable without consideration being given to correction factors for these variables. The methods of applying corrections are also stated.

Consider fuel costs, for example. Messrs. Hirshfeld and Van Duzer assume 15 cents per million Btu. In many localities fuel may be obtained for about 10 cents per million Btu. In such a case, the fuel cost would be much less than that assumed by Messrs. Hirshfeld and Van Duzer for the same station efficiency; but there will be some question as to whether the expected capacity factor of the plant with this low priced fuel would warrant as efficient and as expensive a plant as Messrs. Hirshfeld and Van Duzer assume. If a less efficient and less expensive plant is used, both fuel costs and fixed charges will be affected. At the other extremity, the writer has designed a plant for a foreign location where the fuel cost was about 50 cents per million Btu. Considerable money was spent to secure maximum efficiency from the equipment installed at this plant. A diagram[42] published in January, 1938, shows the remarkable reduction of coal per kilowatt-hour over the last few years. The curves indicate large increases in total kilowatt-hour output with almost stationary total coal consumption.

Referring to investment costs, Table 13[43] gives data on the cost of three different stations. Plant A, situated on the Ohio River, has the lowest coal cost but the highest cost of substructure (probably due to the incidence of floods). Plant C, in a locality with high land values, has also the highest coal costs. Plant B has fuel costs intermediate between Plants A and C.

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[41] Prof. of Mech. Eng., The Johns Hopkins Univ., Baltimore, Md.

[41a] Received by the Secretary July 8, 1938.

[42] *Power Plant Engineering*, January, 1938, Fig. 1.

[43] "Power Supply Economics," by Joel D. Justin, M. Am. Soc. C. E., and William G. Mervine, John Wiley & Sons, Inc., New York, N. Y., 1934.

Note that: (1) Building substructure varies from $4.97 to $15.03 per kw; (2) building superstructure from $8.17 to $21.40; (3) mechanical and electrical equipment from $42.94 to $58.71 per kw; and (4) total cost from $83.05 to $149.72 per kw. Attention is drawn to such actual values to demonstrate the influence of such different factors as location, fuel costs, and foundations upon individual items of plant cost and consequent influence upon total plant cost.

Plant locations are often chosen for reasons that outweigh sub-soil conditions. Thus, Cahokia Station, at St. Louis, Mo., is supported upon very long piles, whereas the Avon Beach Plant near Cleveland, Ohio, is built upon solid rock which had to be blasted away to make room for proper cooling water intakes.

It has been suggested that the total cost could be decreased and the fixed charges lowered by the elimination of the building superstructure. This part of the plant cannot be eliminated entirely for obvious reasons. In certain localities some savings may be effected by this change. In an Australian plant designed by the writer only a roof is provided over the plant to keep off the sun and shed water during a few weeks per year of rain. Outdoor plants are not justifiable in cold climates.

TABLE 13.—UNIT COSTS OF THREE TYPICAL STEAM-ELECTRIC POWER PLANTS

Item No.	Description (1)	Cost,* in Dollars per Kilowatt of Capacity		
		Plant A (2)	Plant B (3)	Plant C (4)
1	Plant capacity, in thousands of kilowatts	70	70	185
2	Number of units	2	2	6
	Cost per Kilowatt of Capacity, in Dollars:			
3	Land, fill, etc.	4.55	2.94	12.60
4	Building substructure	15.03	5.33	4.97
5	Building superstructure	3.17	8.95	21.40
6	Building trim	1.15	0.88	1.78
7	Building equipment	0.45	0.35	2.72
8	Mechanical equipment	24.45	22.38	30.03
9	Electrical equipment	24.90	20.56	28.68
10	Service equipment	7.85	2.80	16.45
11	Substation	4.16	1.40	10.67
12	Engineering and construction	27.12	17.46	20.42
13	Total cost per kilowatt	112.83	83.05	149.72

* Except Items Nos. 1 and 2.

Sometimes plant location and resultant construction cost are fixed by factors other than simply the supply of power. Thus, all plants supplying both power and district heating are located and planned with reference to the heating load. A unique case of the influence of external factors occurs at the power plant in Edmonton, Alberta, Canada, which is adjacent to the city's water-works and filter beds, on the Saskatchewan River. Water is pumped through the surface condensers of the power plant before delivery to the settling basins and filters of the water-works. In winter, vacuum is regulated to provide water at 50° F to 60° F to the settling basins as desired. This water does not cool below freezing in passing through the filter beds and in the pipes under the streets to the various consumers. Although low temperatures are experienced at times, this waste heat from the exhausts of the steam turbines in the electric plant prevents losses from frozen water pipes throughout the city, and even aids in better functioning of the sewers and sewage disposal plant. This system has also been embodied in the new plant now building at Edmonton.

Another factor in the initial cost of plant is the degree of expediency in its construction. Overtime added $8 per kw to the cost of one plant with which the writer was associated, owing to the urgent need of this plant capacity, and union rates for overtime work.

These facts support the contentions of Messrs. Hirshfeld and Van Duzer that every station is an individual problem and that the designer of a power plant must analyze and balance the effect of cost variables before arriving at a design that will produce power for the least total cost.

Even when such a plant is completed the costs secured may be modified by changes in assumed conditions, such as modifications of load curves, changes in fuel costs, inter-connection with other systems, or supersedure, by more economical plant on other parts of the system. For instance, Gould Street Station, in Baltimore, Md., was designed to take a large part of the electric system's base load. However, the subsequent development of the Safe Harbor Hydro-Electric Plant has completely upset the economic assumptions upon which the Gould Street Station was planned.

An analysis of the distribution of costs in the paper for the 400-lb, 600-lb, and 1 250-lb plants, with 40% annual capacity factor and average lower costs, indicates that fuel constitutes approximately 26.2% of costs; labor, 7.5%; maintenance and supplies, 3.6%; fixed charges, 53.7%; and general expense, 9 per cent. The predominance of fixed charges as an element of production cost is evident from such data. Fixed charges may be reduced as already suggested by minimum building structure, simplicity, and standardization of design, by inter-connection, and by avoidance of undue haste.

Messrs. Hirshfeld and Van Duzer indicate that whereas the cost per kilowatt of steam and electric generating equipment has tended to decline, the cost of electrical switching, protective, and transforming equipment has increased, leading to a constant total cost over a period of years. This has been a general experience of plant designers.

It has been suggested that lower plant costs might be secured by the adoption of certain standardized designs for steam plant equipment. Some progress has been made in the matter of standardized turbine sizes and operating conditions in Great Britain, where 650 lb, 850° F, are used widely for plant conditions, with 30 000-kw turbines at 3 000 rpm and 50 000 to 60 000-kw units at 1 500 rpm. Little progress has been made in standardization in the United States owing to the prevalence of superposition, the development of condensing turbines of greater capacity at 3 600 rpm, and the increased pressures and temperatures used in the newer stations. When, and if, these developments are complete, consideration should be given to standardized equipment; but standardization tends toward stagnation. Decreases in the cost of production have followed from pioneering which has always been an expensive undertaking to those who depart from accepted conditions.

Messrs. Hirshfeld and Van Duzer do not consider reserve equipment, which is particularly needed in the case of an isolated plant. When a system has several plants, reserve equipment must be provided in one of these and the cost of maintaining this reserve must be spread over the system output. System inter-connections permit the use of less reserve on an individual system and thus tend to reduce production costs. The authors have confined their paper to production costs only. The public thinks only of the bills they pay. The cost of distribution service and of transmission are usually equal to, or several times greater than, production cost. For instance, the cost of meter rental, meter repair and calibration, meter reading, billing, and collecting is frequently stated as $6 per customer per yr. If this is a domestic customer whose total consumption is 600 kw-hr per yr, the cost of this service alone is 1 cent per kw-hr. In other words, customer service equals or exceeds production cost in the case of many domestic consumers. The consumer's attention should be directed to means whereby these larger items of service and transmission costs may be reduced, rather than to the low charges for production alone. Greater individual consumption by all customers will greatly lower their costs of service.

At the moment, Diesel plants have an advantage over steam plants when small capacities are needed. However, maintenance increases rapidly with age and the useful life is not great.

In the discussion of future possibilities Messrs. Hirshfeld and Van Duzer refer to a new "fuel cell" which is under development. Engineers will be interested in more complete information on this interesting research.

Messrs. Hirshfeld and Van Duzer have presented a clear unprejudiced analysis of the cost of heat-generated energy. This discussion cannot be closed better than by quoting their last sentence: "It would seem that the chances for continued reduction of total power cost at the fuel burning generating plant are indeed very large."

WILLIAM E. RUDOLPH,[44] M. AM. SOC. C. E. (by letter).[44a]—The Second Symposium on Power Costs is a compilation of valuable data and conclusions as applied to electrical energy generation in the United States. The writer feels that one might go a step further with the discussion by carrying it into Latin-American countries, where important power-plant construction continues in progress, in many instances under the direction of North American engineers and with equipment supplied by North American manufacturers. For, whereas power problems of the more densely populated regions of Latin-America have been solved in much the same manner as in the United States, there would appear to be considerable diversity of practice in interior regions where the demand for power comes principally from the mining industry. In one country—Bolivia—for example, two leading mining companies adopted opposite policies during 1937; one arranged to operate its mines and mills almost entirely by water power, relegating existing Diesel plants to stand-by, "because of the high cost of transporting fuel oil and repair parts"; the other installed Diesel units for operating its principal mill, "to secure maximum safeguard against interruptions of service." Under such circumstances, executives of Latin-American mining enterprises, where expenditures for power generally range between 25% and 33% of the total costs, should find good "food for thought" in this Symposium.

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[44] Caracas, Venezuela.

[44a] Received by the Secretary July 8, 1938.

Two important features of the background affecting both capital investments and operating expenses in Latin-American countries are quite different from those in the United States, as follows:

(1) Labor costs are considerably lower. Depreciation of certain Latin-American currencies in terms of the dollar has accentuated this condition in some countries (Venezuela being a noteworthy exception).

(2) Materials costs are likely to be higher, particularly on manufactured products and fuel, because of transportation expenses and import duties.

In mountainous regions away from the coast, these factors contribute to make the comparison between hydro energy on the one hand, and steam or Diesel energy on the other, more favorable to the former; and yet, even here, the two forms of generation are complementary rather than competitive.

A more recent example of hydro-electric plant construction costs is the Kilpani Station on the Rio Yura, in the southwestern part of Bolivia. This installation, with a firm capacity of 4 000 kw, was placed in operation during May, 1937, to supply current to mining establishments at Potosi and Pulacayo, and is typical of plants in mining districts where relatively small tonnages of ores of higher priced metals (in this case, tin and silver) are treated. It utilizes a head of 985 ft, having slightly less than 2 miles of canal, and 4 000 ft of penstock. Its unit cost of $74 per installed kw, exclusive of transmission lines, was divided as follows:

Description	Cost	Percentage of Total
Dam	$ 2	3
Waterway	36	49
Power-house and equipment	27	36
Roads, etc.	8	11
Land and water rights	1	1

All these unit costs are lower than the costs for the composite of seven higher head plants tabulated by Professor Barrows, except that of road work which is four times as large. This outstanding item reflects ruggedness of terrain and inaccessibility of the region in which the plant was built, factors which hold not only in most parts of Bolivia, but also throughout the Andes.

Obviously, lower costs of dam and waterway, as compared to those in the United States, were due to lower wages, which at official peso exchange in Bolivia at that time averaged about 35 cents U. S. currency per day for peons, and about 75 cents for skilled labor. The dam was earth with a clay core, and the canal was masonry and concrete with rock and sand convenient to the site; and these items benefited most from such labor rates. That power-house and equipment costs were lower than in the United States was due to a temporary condition whereby machinery was purchased from German manufacturers at extremely low prices, through the use of export marks. However, inability of these manufacturers to guarantee early deliveries at the present time, and unfortunate experiences with some of the equipment which they have supplied, would indicate that North American manufacturers have in no way lost their South American market. Costs of land and water rights were low because the site was in a region of 14 000 ft altitude inhabited only by Indian communities.

Attendance and maintenance expenses of operating water-power installations in Latin-America are generally lower than in the United States, again because of the lower scale of wages. However, the item of depreciation in the case of plants furnishing power for mining enterprises, must include consideration of the depletion of ore reserves, causing ultimate loss of load. On the other hand, operators of steam and Diesel installations away from the coast must cope not only with first cost of oil or coal, but also with high transport expenses from the nearest seaport. For the greater part mines are located at elevations of 9 000 to 16 000 ft in the Andes, and the crustal movements which here caused rock structures to become impregnated with veins of metals also provided the steep gradients which permit power to be generated from falling streams for extracting such metals. Rise and fall matter little to a high-tension line, but the toll which such rise and fall exact upon the delivery of oil or coal to remote spots in the Andes represents no small item of the cost of producing Diesel or steam power.

The hydro-plant installation in mining regions of the Andes often has the disadvantage of being remote from the center of operations, entailing difficulty of access for replacements or repairs. Hence, unscheduled interruptions, while less frequent than at Diesel and steam plants, are likely to be considerably more costly because of time losses. Destructive landslides and floods sometimes result from heavy showers in these regions of seasonal rainfall, and abnormal droughts also occur at long intervals. Against such emergencies and transmission-line interruptions, enterprises having only a single source of water energy find it advantageous to install additional Diesel or steam units, or, as is more often the case, to hold in reserve the Diesel or steam units which probably preceded the hydro-plant. Most feasible, economically, is the combined installation in which the Diesel or steam units carry only occasional peak loads during normal years when there is plenty of water, but where they carry the base load during dry years when it is necessary to conserve storage for the minimum stream-flow period just prior to the next rainy season. Inter-connected systems are rare in mining regions, due to obstacles of distance; a recent interconnection involved the installation of frequency changing equipment.

Investments in power plants so large that the output cannot be absorbed immediately are rare in Andean mining regions, because of the high rate of return upon capital required for projects in foreign countries. Much of the expansion in South American mining enterprise during more recent years has been paid for out of current earnings, and with the application of this practice to power plants, the large project is likely to be deferred, while a group of smaller projects, although less advantageous in the aggregate, might proceed. These larger projects may not count upon Government aid, as part of public works programs to reduce unemployment, most Latin-American countries lacking both the money and the need for such expenditures.

Louis Elliott,[45] Esq. (by letter).[45a]—The analysis presented by Messrs. Hirshfeld and Van Duzer gives as good a view of thermal-plant energy cost as is possible within the limits of a short paper. With the assumptions made, the stated range of energy production costs, as summarized in curve form in Figs. 2, 10, 13, 14, and 16, gives a reasonable idea as to what is to be expected from steam and Diesel stations in the United States. General considerations, and detailed factors bearing on energy costs, are well covered by the authors. It is particularly desirable to call attention, as is done in the paper, to the magnitude of general overhead—assessed at from 10% to 25% of specific production costs and fixed charges. These overheads, discussed under Group 3, are summarized in the "Introduction."

Engineers studying and utilizing the data presented will doubtless keep in mind that the costs cited cover a wide range with respect to the assumptions made, but apply, as to investment basis, to only one unit cost of plant—$100 per kw of capacity. Therefore, in considering energy costs derived, allowance must be made for spare capacity, which is necessary on every power system, as well as for any variation of unit investment, under actual conditions, from the $100 assumed as a round figure.

Papers of the character presented by Messrs. Hirshfeld and Van Duzer—and this remark applies to sections of the Symposium covering hydro-electric as well as thermal-electric stations—are of general educational and historical value, but should be used with caution as a guide in studying any specific problem. It can scarcely be too strongly emphasized that in analyzing a specific case and determining the economic type and size of a power development, only general guidance can be obtained from a broad study of this kind, and that the most careful analysis of special conditions obtaining, detailed estimates of costs and economies, and broadest judgment, must be brought to bear before a proper decision can be reached. Such a study, for any given case, calls for a command of the best information with regard to modern power equipment and systems, and familiarity with the technical tools of study, such as the various forms of load and duration curves.

The "art" of designing and operating a power system, whether it be supplied by hydraulic or thermal plants, or by a combination of the two, has become a quite highly developed specialty, and analyses and decisions with respect to power development should not be entrusted to men who have not, through years of training and experience, become thoroughly conversant with the body of data and practice that has been developed. Investigations and reports with respect to power systems or developments, founded upon elementary conceptions and incomplete data, or made on an emotional basis, may well arrive at erroneous conclusions.

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[45] Cons. Mech. Engr., Ebasco Services Inc., New York, N. Y.

[45a] Received by the Secretary July 11, 1938.

H. G. GERDES,[46] M. Am. Soc. C. E. (by letter).[46a]—All opinions advanced and estimates made are entirely those of the writer, and all data cited have been taken from published sources. Statements made herein have neither been approved nor disapproved by the Federal Power Commission.

Natural Basis of Complementary Use.—The principal thesis expressed by Mr. Whitman—namely, that the cost of power from combined steam and water power sources is frequently lower than from either source alone—is a paradox, the answer to which, unfortunately, does not appear to be quite clear to the public at large.

In view of recent apparent controversy between advocates of steam and water power, the fundamental principle cannot be emphasized too often that steam and hydro-power sources are more often naturally complementary rather than competitive. This affinity proceeds out of the extremely varying public demand for electricity, whereby engineers are enabled to couple unlike sources of power together in varying degrees to meet this demand with greater economy than from either source alone.

An exception to the rule of complementary economy of unlike power sources occurs where a constant power source such as Niagara Falls attracts a constant power consumption load of like nature. Exceptions, not to the rule, but to the practice of incorporating a steam plant as one of the unlike power sources, occur in the Pacific Northwest where base-load, low-head hydro-plants on the larger rivers may be combined with peak-load, high-head storage hydro-electric plants on near-by mountain streams.

Phases of Combined Power Use.—Growth in the development of combined power use has been marked by many changes, generally characterized, however, by exhibition of the complementary function described. In California, steam plants have successively served major hydro-systems as load center stand-by capacity, peak-load capacity, and latterly as base-load carriers. In the Northwest, steam plants have been constructed on hydro-systems as peak-load carriers, as stand-by reserve, and as quickly added increments of capacity. In the Southeast, steam plants have largely functioned as stand-by reserve and intermediate incremental construction on systems that are predominantly hydro-electric.

In the Middle Atlantic section of the United States, where factories operated by steam power were common before the opening of the electrical era, the growth in central station steam capacity by incremental steps has been a natural evolution, and the complementary advantages of hydro-power have been realized only in recent years.

Factors Favoring Combined Operation.—Peak-load hydro-plants have been more often economically feasible in large sizes of 50 000 kw and upward. Low unit cost demands a large installation, which may be so greatly in excess of the reliable stream flow as to be available only a few hours per week. Interconnection of utilities into large systems increases the usefulness and economic feasibility of such peak-load hydro-plants both by allowing large incremental steps in new capacity and by permitting hydro-peak operation on a higher and sharper weekly load peak in which the same number of stream-flow kilowatt-hours available may be utilized in a shorter time, making firm a larger capacity.

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[46] Engr., Federal Power Comm., Washington, D. C.

[46a] Received by the Secretary July 12, 1938.

Quite properly, Mr. Whitman remarks that the cost of furnishing peak power by steam is expensive. A renewal of search for peak-load, hydro-plant sites may be expected as soon as load growth and super-position complete the conversion of certain existing obsolete steam plants into base-load carriers.

Combined Hydro-Steam Energy Cost.—Two fairly typical illustrations, drawn from the operations of hydro-steam systems, are presented herein. In these systems the total load is large and the annual peaks both high and sharp. The load will absorb without waste, large blocks of river-run fuel saving power; hydro installations with capacities as great as ten times the low stream flow have been made, and the determining factor is usually the kilowatt-hour storage available to firm the plant capacity on the system peak.

It will be understood that although the data given are believed typical and fair, many simplifying assumptions have been made to bring the examples within the bounds of a simple comparison for the purpose of this illustration.

TABLE 14.—Typical Eastern Hydro-Steam; Assumed Capital and Annual Charges on Dependable Capacity

Item No.	Description	Capital cost, in dollars per kilowatt	Annual Cost								Total
			Fixed					Operating			
						Total		Operation and Maintenance			
			Percentage return on investment	Taxes and insurance, in percentage of total cost	Percentage depreciation	In percentage of total cost	In dollars per kilowatt	Constant, in dollars per kilowatt	Variable, in mills per kilowatt-hour	Fuel,* in mills per kilowatt-hour	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1....	Hydro-electric power plant	200	6	1.5	1	8.5	17.00	1.60	....	....	18.60
2....	Transmission and reception	40	6	1.5	1	8.5	3.40	0.40	....	....	3.80
3....	Steam power plant.......	120	6	2.5	3	11.5	13.80	5.00	0.30	2.25	18.80+2.55

* $4.50 coal at 1 lb per kw-hr.

The cost data are comparative and do not include items not affecting the comparison, such as general overhead, etc.

Example 1.—On a large Eastern combined power system, a peak-load hydro-plant of one-third the total system capacity operates at about 20% load factor (1 750 kw-hr per kw-yr) to cover the system weekly peak load.

Additional stream flow, in excess of required peak-load storage, averages 3 600 kw-hr per kw of installed capacity annually. The annual load factor of the system is about 50% and the resulting annual load factor of the steam plant is 44.5 per cent. Typical fixed and operating costs have been assumed, as shown in Table 14 for the purpose of illustrating the cost of combined energy generation and delivery to the load center as compared with the cost of serving an independent portion of the same load by either means alone. The steam station is assumed to be at the load center.

From Table 15(*a*) the comparative cost of combined steam and hydro-energy delivered at the distribution network is seen to be 6.24 mills per kw-hr (Item No. 10) as compared with 6.85 mills (Item No. 7) for the entire steam generation, resulting in a saving of 9% (Item No. 11). On a 3 000 million kw-hr annual output system, the indicated saving of 0.61 mill amounts to the substantial sum of $1 830 000 per yr.

Based on the following plant and load data, the typical load factor diagram in Fig. 33(*a*) shows the position on the annual load diagram of the three classes of power available:

Hydro-plant peak-use factor $= \frac{1\,750}{8\,760} = 20.0\%$

Hydro-plant fuel-saving use factor $= \frac{3\,600}{8\,760} = 41.0\%$

Total hydro-use factor $= 61.0\%$

Steam plant use factor $= \frac{7\,790}{8\,760 \times 2} = 44.5\%$

System load factor $= \frac{1\,750 + 3\,600 + 7\,790}{3 \times 8\,760} = 50.0\%$

Hydro-plant energy $= \frac{5\,350}{8\,760 \times 3/2} = 40.8\%$

Steam-plant energy $= \frac{7\,790}{8\,760 \times 3/2} = 59.2\%$

Total energy 100.0%

The three classes are as follows:

(1) Firm hydro-energy rendering the installed capacity firm on the weekly load curve of the system (assumed at 1 750 kw-hr per kw, or 20% plant factor);

(2) Excess hydro-energy available during months of high stream flow, carrying the base load during the time available (assumed as 3 600 kw-hr per kw per average yr); and,

(3) Steam energy carrying base load and semi-peak during months of high stream flow (assumed as the 7 790 kw-hr per kw-yr shown on the annual load curve of the system).

The foregoing substantial saving is made although the hydro-investment is $240 per kw as compared with $120 for steam capacity.

Example 2.—On a somewhat smaller Eastern system in which conditions are similar except that hydro-capacity cost is only $145 instead of $200 per kw, with an operating cost of $0.80 per kw, the comparative cost of combined power at the distribution network is 5.79 mills per kw-hr, a saving of 16% through combined energy generation. The incremental saving of 1.06 mills amounts to $2 120 000 annually on a 2 000 million kw-hr system.

TABLE 15.—COMPARATIVE COST OF COMBINED ENERGY GENERATION

Item No.	Description	Capital cost, in dollars	Percentage of capital cost	Cost of Power			
				In dollars per kilowatt-year	Mills per Kilowatt-Hour		
					50% load factor (4 380 hours)	61% load factor (5 350 hours)	44.5% load factor (3 900 hours)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	(a) TYPICAL EASTERN HYDRO-STEAM SYSTEM						
	Hydro:						
1	At switch-board	200	9.3	18.60			
2	Transmission and reception	40	9.5	3.80			
3	Power loss in transmission (10%)	...	...	2.25			
4	Total, ready to distribute*	240	10.3	24.65	§	4.60†	
	Steam:						
5	Fixed components	120	15.7	18.80	4.30		4.82
6	Variable components	...	...	...	2.55		2.55
7	Total, ready to distribute*	120	...	...	6.85		7.37
	Combined Power:						
8	Hydro: 40.7% × 4.60	...	...	...	1.87		
9	Steam: 59.3% × 7.37	...	...	...	4.37		
10	Total, ready to distribute*	...	...	...	6.24		
11	Percentage of saving through combined use	...	...	...	9.0		
	(b) HIGH-HEAD AND LOW-HEAD HYDRO-SYSTEM						
	Low-Head Hydro:						
12	At switch-board	104	5.8	6.00			
13	Transmission and reception	30	7.0	2.11			
14	Power loss in transmission (10%)	...	...	0.81			
15	Total, ready to distribute*	134	6.7	8.92	2.04	1.37†	7.6‡
	Peak-Load Hydro:						
16	At switch-board	60	6.2	3.71			
17	Transmission and reception	30	7.0	2.11			
18	Power loss in transmission (10%)	...	...	0.58			
19	Total, ready to distribute*	90	7.1	6.40			5.46‡
	Combined Use:						
20	Low-head hydro, 1.37 × 89.3%	...	...	...	1.22		
21	Peak-load hydro, 5.46 × 10.7%	...	...	...	0.59		
22	Total, ready to distribute*	...	...	...	1.81		
23	Percentage of saving through combined use	...	...	...	11.0		

* Includes only the comparative items stated. † 74.4% load factor (6 520 hr). ‡ 13.4% load factor (1 170 hr). § Only 20% load factor hydro firm.

Complementary Hydro-Systems.—The advantages of combining power sources of inherently different characteristics in one system are by no means limited to the large fuel-fed networks of the East. Although only a few have been built thus far, a number of opportunities exist for the complementary operation of base-load run-of-river and peak-load storage hydro-plants in the same system. Some of these complementary operations, however, are subject to the economic and operating disadvantages of connecting transmission lines more than 100 miles long to the mutual load center.

Power from low-head plants on Western rivers may be combined with peak-load hydro-power from high-head storage plants, and it appears of interest to study the cost of possible combined operation. Large sustained river flow would permit a high plant utilization factor at these low-head installations. Deficiencies in power output (which are ordinarily short but fairly severe) ranging from 10% to 50%, would usually occur during the summer flood period of reduced head and winter freezing periods of low water. During the winter low water, rated capacity would not be impaired, but operation at a plant factor less than 100% would become necessary.

Due to the low average head, the cost of power plant and machinery is relatively higher, and the cost of providing peaking capacity would be, therefore, greater than at higher head plants. The high utilization factor, and the large proportion of total cost required for power-plant building and machinery, appear to "earmark" these low-head river-run plants as natural base-load power producers. Operation on a 50% load factor, for instance, would require wastage of nearly one-half the power of the river otherwise within the plant capacity. Conversely, combined operation together with a peak-load storage plant would enable the fullest use of available power and greater earnings upon the larger unit investment necessary in the low-head plant. The function of the high-head plant would be to carry all peak load in excess of the low-head plant capacity and all seasonal capacity (winter) or energy (summer) deficiencies of the latter.

Example 3.—As an illustration of the advantages of combined use a study of possible combined high and low-head operation is presented herewith on the basis of the following data:

Peak hydro use factor $= \dfrac{274}{0.4 \times 8\,760} = 7.8\%$

Peak hydro deficiency use factor $= \dfrac{195}{0.4 \times 8\,760} = 5.6\%$

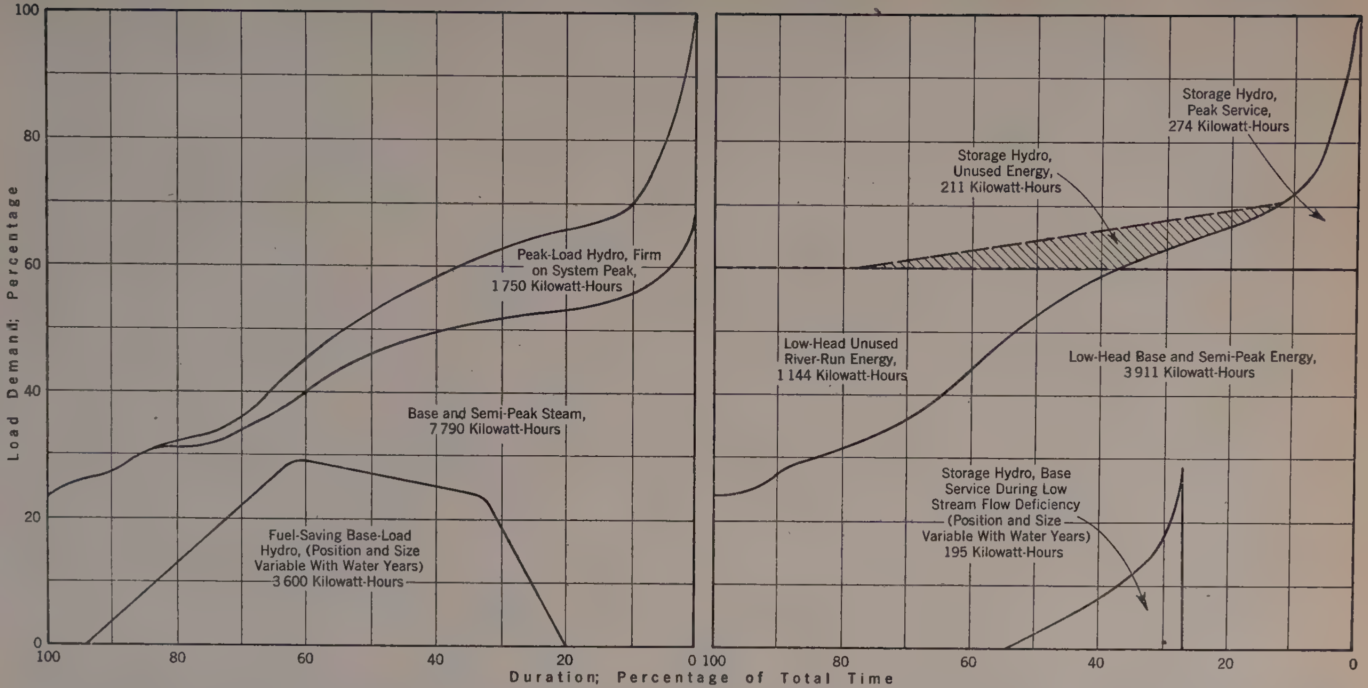
Total peak hydro use factor $= 13.4\%$

Low-head hydro use factor $= \dfrac{3\,911}{0.6 \times 8\,760} = 74.4\%$

High-head plant: Peak + deficiency energy $= \dfrac{274 + 195}{8\,760 \times \frac{1}{2}} = 10.7\%$

Low-head plant: Base energy $= \dfrac{3\,911}{8\,760 \times \frac{1}{2}} = 89.3\%$

Total energy.. 100.0%



(*a*) Eastern Hydro-Steam System; Allocation of Hydro and Steam on Annual Load Curve (Diagram Represents 3 Kilowatts at 50% Load Factor).

(*b*) High-Head and Low-Head Hydro; Allocation of Peak and Base Hydro on Annual Load Curve (Diagram Represents 1 Kilowatt at 50% Load Factor).

FIG. 33.—TYPICAL COMBINED LOAD-FACTOR DIAGRAM

The following assumptions as to cost and operations are made:

(1) Low-head power-plant cost, \$104 per installed kw with an installed cost of transmission and reception facilities at \$30 per kw;

(2) High-head storage development with 1 700 kw-hr per kw-yr of available energy, at an installed cost of \$60 per kw plus transmission and reception facilities at \$30 per kw;

(3) Interest 3.5% and amortization through a 40-yr, 3.5% sinking fund; and

(4) Combined operation on a 50% load factor similar to Fig. 33(*b*).

Fig. 33(*b*) illustrates the complementary function of the peak-load storage hydro, both in carrying 40% of the combined annual peak load and in operating on base-load from storage during the winter period of low stream flow. The comparative cost of power at the distribution net is calculated at 1.81 mills (Item No. 22, Table 15(*b*)) for combined use as compared with 2.04 mills (Item No. 15) from the aforementioned low-head plant, a saving of 11% (Item No. 23). With an average annual output of 3 000 million kw-hr, the 0.23 mill saving would amount to \$690 000 per yr.

Unused hydro-electric energy shown as the shaded area at the left in Fig. 33(*b*) might be utilized by raising the plant factor, either by developing a higher load factor or by combination with additional semi-peak generating capacity.

General Comments.—The combination of power sources of unlike characteristics, as described herein, so as to fit a system load shape best, represents (in common with load-factor increase through diversity in use) a mutual reduction in reserve capacity. This is one of the advantages of grouping power systems through inter-connections over natural market areas.

LEVERETT S. LYON,[47] ESQ. (by letter).[47a]—In their public aspects power and the costs of power are subjects of great consequence to economists. Mr. Uhl has given an excellent discussion of these costs. He has enumerated the important items, both fixed and direct, and has emphasized the significance of each. He has indicated that the engineer must not overlook them in analyzing projects—that if he does he changes from an engineer to a promoter.

Although it is impossible to overstate the importance of costs from the point of view of constructing projects, it is from another point of view that they become of great concern in public policy, and to economists. The economist's interest in cost, in engineering, and in any other matter, is essentially in terms of public policy in relation thereto. It is the failure to realize that the economist is concerned with public policy which transcends the limits or boundaries of any given industry, that causes many people to say that economists never agree. Such a position is untenable. It comes from confusing so-called economists with those who are what some like to refer to as professional economists. There is a difference between the farm economist, the labor economist, the business economist, the practical economist, and those who more truly represent the profession. The difference is that the latter type are interested in the nation's welfare rather than the welfare of any particular industry or group.

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[47] Executive Vice-Pres., The Brookings Inst., Washington, D. C.

[47a] Received by the Secretary July 12, 1938.

The professional economist is concerned with the question of how to use the nation's resources to the best advantage in the public interest. This means securing the largest possible output for the smallest input—a good engineering concept. In more detail, this means that if input can be had at a lower cost in one form than in another the lower cost form is the one to use.

Questions of cost thus viewed become questions of public policy and, therefore, of interest to the economist as costs find reflection in prices. In ordinary business, such as manufacturing or merchandising, or in any other competitive field, the costs of an individual company may have no direct relation to the prices at which it can sell its product. If there are five manufacturers of a given commodity for example, competing in a given market, and each has a cost that is different from the others, the one with the lowest cost will get as much of the business as he desires or can supply—unless the others sell below so-called "costs." It does the high-cost competitors no good to complain that their costs will not permit them to compete profitably. Prices simply are not made that way in competitive markets. In such a situation a high-cost manufacturer has no choice but to forget part of his costs, to close his eyes to the nicely worked-out "cost system" of his accountants, and to remember that competition came before accountants and will live long after them unless the nation becomes essentially socialistic.

In such a situation the manufacturer "closes his eyes" to some of his fixed charges. This is not difficult if his fixed charges are not bonds or other borrowed money on which there is a definite interest charge. If there is no such charge he will come to the conclusion that his investment is not worth as much as he thought it was—that it is indeed only worth what he can earn on it.

If he has a fixed charge in the form of bonds which leads to a foreclosure or a re-organization, his fixed charge after the re-organization will no longer be what it was before. By the ritual of a Court procedure or of a re-organization committee his costs will have been reduced. Accordingly, if he must think of his prices in terms of cost, all is now made easy. Although the original investment in the company has not been changed, the bookkeeping can now be changed so that "book costs" may now be in line with selling prices.

Of course, there is a limit below which prices will not be reduced even in a competitive market. In general, a manufacturer will hesitate to go below his direct charges although he may be quite willing to conclude that anything he can make above them is better than nothing. Many manufacturers find it profitable to go even below their direct charges at times in order "to keep in the game during a period of sharp recession." It may pay well to take a loss even on direct charges for a time with a view to being "in the picture when the situation is better."

The idea that, in all industry, prices should cover costs is persistent. The business man finds it difficult to escape from the feeling that "my investment is in this business, I am entitled to a return." This seems an entirely natural point of view. Indeed, it is a natural reaction to the situation, but there is an important idea of great social significance which opposes it. This idea is the one already stated, namely, that the resources of the nation should be used so as to give the greatest possible output at any given time for the smallest possible input. This really means that a buyer, whether he is a consumer buyer, or a business buyer, should be in a position to purchase, at any time, at as low a price as any one will offer at that time. A consumer who buys a hat should be able to purchase it at the lowest price for which any one is willing to make that kind of a hat. A power company should be able to buy its coal, its machinery, construction materials, and to borrow its money at as low a rate as any one at any given time is willing to furnish those things. No one has ever yet presented another plan so well calculated to make the resources of a nation at all times give the largest total yield to all the people as this plan of economic organization.

Alternative plans that have been presented have certain merits. Their fundamental weakness is that without such a measure as a competitive market gives, of what the price should really be, there is no satisfactory way of knowing what it should be. For example, to claim that prices should cover costs leads into the greatest difficulties. Whose costs, for example? On such a basis (that a manufacturer is entitled to his costs) there would be no incentive to keep costs down. Indeed, if he believes he should have a profit as a percentage of his cost, higher cost would be an advantage.

In the field of power, however, the competitive method of price determination will not apply. The reason is fundamentally the same as the reason in favor of competition in other fields. The people cannot afford to get along without competition in most fields; they cannot afford to have competition in this one. In other fields, without competition, goods cost too much. In the power field, with competition, goods cost too much. It has been discovered long ago that competition in the production of power for communities (and of many other commodities which are produced by so-called public utilities) will either result in building so much equipment that a charge based on costs will be higher than is necessary, or that the competitors will form a monopoly and charge an extortionate rate or that, in fear that competition will cut prices so low that the return on investment will be less than it would be in other fields, the desired facilities will not be built. In other words, in the field of power there is what economists call a "natural monopoly."

Since a monopoly cannot be trusted to create prices in the public interest, some social regulation of rates has been established in every State. A social regulation is directed primarily at the question of what price should be charged. Since the commissions which have been given this responsibility have no competitive tests they have groped, none too happily, in an effort to find a satisfactory method of relating costs to prices. This is not the time nor the place to describe the trials and tribulations that have beset the public utility commissions. Every one knows something of their struggles to determine what costs should mean—not in terms of expenses for building a project, but as a basis for rate-making. Shall they take all costs into account? Will all costs mean production costs? If so, production when? Will production mean original production costs or will it mean reproduction now? Or will it mean that beautiful but vague concept, cost on the basis of prudent investment?

All Mr. Uhl's observations on variables are relevant for consideration in trying to relate cost to prices. One good example is his comment on obsolescence. If one wishes to compute the cost of producing power as a basis for determining the prices at which it should be sold one would need to include a charge for obsolescence; but this, in turn, as Mr. Uhl has shown, depends upon the calculations which one makes as to how rapidly new equipment (even new inventions) may come into the field to supplant the old. Even if everything which the past tells is included, something new may happen. There may be an Edison, and coal may be made relatively obsolete in many communities. Gas may replace electricity. Some physicist may "pull" a power ray out of a laboratory which will make hydro-electric plants old-fashioned almost over-night.

What happened to the canals when the locomotive was invented is a matter of history. The struggles of the railroads and the Interstate Commerce Commission in an effort to find a rate for railroad traffic that will take care of the costs of the railroad companies is a problem of the present day. They are now at that nightmarish stage at which no one seems to know—even if they wish to cover all costs—whether it will be better to push the rates up or to push them down. "Verily, the way of the price fixer is hard" even when he does the fixing with the aid of the law.

Quite aside from these deeper issues, however, there are others which are of public importance when prices must be related to costs. It is always proper to ask whether the costs are fair or unfair. Are they really the necessary costs or are they "padded"? Is the enterprise capitalized at the minimum necessary to furnish the service or is it over-capitalized? Are officials over-paid or otherwise? To what extent are holding companies and similar financial and managerial mechanisms an effective means of organization and operating, and to what extent, if any, are they means of adding costs, positions, and salaries, on which rates are to be computed? Such questions should properly be asked and answered, not in an atmosphere of acrimony and controversy, but of careful consideration of the accounting, engineering, and economic issues involved.

It is the raising of such issues of costs as these which has introduced the now much discussed "yardstick" as a proposed method of determining what prices should be. Of course, the so-called "yardstick" is not a new idea. Every method that can be applied to determining what prices shall be—even competition—is a yardstick; but the great extension of Government operations as a means of comparing costs does offer, in size at least, something new. It also introduces new difficulties by the half-dozen if not by the score. To such problems as how to measure costs, reproduction costs, and the like, it introduces that most confusing of all cost concepts, joint costs. To illustrate: If a dam is built as a means of creating power and, at the same time, as a means of providing irrigation, and, at the same time, for purposes of flood control, and, at the same time, to lift a backward community into an industrial community, how much of the costs of the project—of each item of fixed costs—such as Mr. Uhl mentioned, should be charged against the cost of power?

Finally, Mr. Uhl's excellent paper refers to the contributions of the power industry to the American nation. As indicative of these contributions he cited the investment and the contribution through taxes which had come from this industry. These criteria of contribution are worthy of mention, but they do not do justice to the power industry. Although it is politically helpful to point out the tax payments of a public utility, its real contribution to the nation must be measured in quite different terms.

Man's effort, from the earliest recorded time, has been to increase the output which he can secure from his input in labor and toil. It is as the power industry has added to this output or lessened this input that its contribution has been made. Mankind has always had resources at his disposal. He has progressed on power—as he has been able to utilize natural forces to supplement or replace his own puny efforts. It must be in calculations of the additions—or, perhaps, better—the multiplications of Man's unaided power which have come through the power industry that one can best estimate what it has given to the world. It has added a series of problems, and some of them are difficult, and it has not been done without cost, but the credit balance for the power industry is an outstanding one.

JOHN T. MADDEN,[48] ESQ. (by letter).[48a]—One unfortunate circumstance in the economic life of this nation is the frequent necessity of attempting to settle important questions by steering a middle course between two evils. In order to avoid Scylla, or the evil of monopoly, on one hand, and Charybdis, or the evil of unrestrained competition, on the other hand, legislators pursue the middle course, and public utilities become a regulated monopoly. This middle course does not always mean "clear sailing." In avoiding twin evils, other vexing difficulties are often encountered. To shift metaphors, a veritable witch's broth has been brewed in this public utility stew, combining engineering, economics, finance, law, and politics, and there are some who say that too little of that important condiment—ethics—has been used.

There is no doubt that an incalculable amount of economic waste could have been avoided if politics had been kept out. Better cooks would have left out the complexities, uncertainties, and paradoxes of the law. Sound principles of economics, combined with ethics, would have avoided such dishonest and shoddy financing as there was. The engineering contribution seems to have been sound; at least, there is little condemnation on this score. However, although perfection in this area of human activity seems as unattainable as in others, on the whole, the results are fairly satisfactory; accomplishment is not as poor as some have alleged.

The story of this industry is the same old tale of industrial development in the United States, and it follows the usual pattern. A new industry produces a service which later comes to be regarded as essential. Progress is slow at the outset, proceeding in lame and halting fashion. For a time the venture is highly speculative, but, nevertheless, it manages to attract to itself the capital, skill, and energy of those venturesome souls always present in an acquisitive society. The industry reaches its "adolescent" period, making the customary errors of youth. Ultimately, it attracts the attention of politics and the law steps in. Controversies arise, and instead of resolving them with the aid of ethics and economics in simple and direct fashion, differences increase. The penalty for economic success is to have one's "head chopped off" in appropriate "Alice in Wonderland" style.

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[48a] Received by the Secretary July 12, 1938.

Despite the controversy and criticism of regulation and rate-making, the summary data in Table 12 are most impressive. The wonder really is that such progress could have been made in view of all the difficulties placed in the path of the electric light and power industry. Mr. Uhl concludes his paper with the statement that the "power industry is of increasing importance to the welfare of the nation and has made tremendous contributions to its wealth and income." This is a temperate statement, appropriate and becoming in the engineer. The record of progress from 1912 to 1936 is merely indicative of the potential capacity of the industry to contribute to the future wealth, income, and social betterment of the nation, if legitimate expansion could proceed. In fact, the record cited serves to "highlight" the serious damage to the national economy and welfare wrought by those who have used the industry as a political football. Mr. Uhl did not stress the fact that the power industry is a most important arm of the national defense, and those whose aim is to wreck it, or whose actions impede its capacity to serve the nation in the event of war, will have much to answer for at the bar of public opinion.

This Symposium has demolished the pretensions of those who have supported the alleged value of "yardstick" rates established by public plants scattered about the nation. The variable factors are so many and so incapable of being reduced to a common denominator that any comparisons would be worthless.

In his discussion of rate theories,[49] Professor James C. Bonbright has stated that "whatever the proposed remedy, the disease to be cured is generally diagnosed as one caused by the focal infection of the fair value doctrine," which he characterizes as Court-made law. The writer does not subscribe to his view. Despite the criticism of the fair value doctrine, relatively few rate controversies have reached the Courts. The legislature imposed regulation; and, to continue the medical metaphor, the regulatory "treatment" was to be administered to "patients" or "cases," no two of which, as shown by this Engineering Symposium, are alike biologically, chemically, or physically. While the legislature developed its "science of medicine," it was left to the commissions and the Courts to practice "the art of healing." There is a vast difference between the science of medicine and the art of healing. In this Symposium, the engineers who have studied the cost of power generation have shown that, although there is a certain general uniformity in the "anatomy" of physical plants, they vary considerably. The economist knows that the investment costs—"the flesh and glands"—vary far more than the "anatomy." In establishing the fair value doctrine, the Court left leeway for varying and individualistic treatment of the "patients." The physician who prescribes only one remedy for all his patients does not command much respect.

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[49] "The Valuation of Property," by James C. Bonbright, McGraw-Hill, 1937, Vol. II, Chapter XX, p. 1079.

Proficiency in the art of regulation has improved greatly, just as the art of healing has shown great advance. Despite the "focal infection" which Professor Bonbright finds, the pragmatic test of the Court-made law reveals that the "patient" got along fairly well, although alternative treatment was not employed. The "doctor's" expenses, such as appraisals, legal and engineering services, and Court costs, were not unduly expensive considering the financial circumstances of the "patient." Furthermore, before 1932, this growing industry had made significant advances in its engineering technique; it extended the benefits of its service to countless hamlets and many thousands of persons who have not even yet had the benefits of railway service; it raised at reasonable cost all the capital required for its development; the cost of that service was greatly reduced; and, a sound basis has been laid for still further progress if the politicians will cease harassing the industry. Abuses have existed, of course; "write-ups" and "write-downs" have occurred on the books, but despite the allegations of watered capitalization, so commonly made, the fact is that no one knows either the prudent investment cost or the fair value of the existing plant, and no one will know until an inventory and an appraisal are made.

Economists would find it difficult to point to any other business institution that has made such forward advance in so short a period, with equally beneficial social consequences. In fact, if social engineering were half as proficient as civil, electrical, and mechanical engineering have been, there would be no power issue of any consequence to-day.

Whatever may be the outcome of the prudent investment or fair value doctrines, or whatever may be the outcome of "yardsticks" and "big sticks," more attention is to be given to the preparation and maintenance of continuing fixed property records. Engineers will devote more attention to, and perfect, their techniques in measuring the effect upon fixed property of the several causes of depreciation. Assume that the engineers accomplish all that is to be desired in these matters; assume that a technique for measuring the extent of wear and tear has been developed with reasonable accuracy; and, assume that engineering experts develop a foresightedness as to obsolescence—there is still one more question to be answered. Is the object of the business struggle to preserve intact the mere dollar capital invested, or shall it be to protect the "real" invested capital in terms of purchasing power? The distinction is important and this problem is no less baffling than other problems connected with this difficult question. The gyrations of price levels in the United States since the World War vitally affect the accounting process, which attempts to record financial facts in terms of a variable "yardstick." It will be observed that the computation of depreciation under either the fair value principle or the prudent investment principle does not touch upon this point, particularly if by the latter principle is understood that, in so far as depreciation is concerned, the sum to be charged to operating expenses is the proportion of mere dollar investment that is to be amortized.

Recent experience bears out the suggestion of Mr. Scharff that in competitive industry it will be necessary or desirable to have a more accurate determination of the monetary sums to be set out in the accounts for the various causes of depreciation. In too many instances the depression beginning in 1929 revealed an impairment of dollar capital through neglect in this respect.

There is another cause of depreciation beside those commonly mentioned in discussions on the subject; namely, shifting in population, or shifting of industries, which may completely alter the character and extent of a company's business, and increase its obsolescence factor. The perfection of air-conditioning may have a serious effect upon the popularity of seaside resorts in the summer. There still remains much work to be done in a more accurate study of the market, and the results of such a study might indicate an increase or decrease in the provision for obsolescence. Apparently, Mr. Scharff recognizes the difficulty of shifting the loss due to obsolescence; it may be placed upon the present generation of rate payers, upon a later generation; or it may fall upon the owners of the enterprise. In some cases it may be shifted to employees who will accept reduced wages to compensate for the loss to an employer who finds himself in an unfavorable competitive position with other manufacturers with more modern equipment.

In 1906, Woodrow Wilson stated that the automobile would be responsible for an increase in socialism since the vulgar display of the wealth of those who owned automobiles would create dissatisfaction. Although this fear proved groundless, the automobile did injure the revenues of railroads and electric railways. This Symposium displays no present fear on the part of engineers that radical changes in the art are in prospect. One can easily dream of a revolution that would jeopardize the entire investment in electrical properties. Stranger things have happened. If more extensive public ownership is to come eventually, the loss would then fall upon taxpayers. Perhaps taxpayers should think more than once before they rush heedlessly into Government ownership.

Briefly, then, it appears that engineering science has so definitely established comparative costs and uses of hydro and steam projects that Government and finance should be governed by these conclusions. Further progress in engineering studies on depreciation and obsolescence will determine the final disposition of this vexing question, but the writer concludes that, in the future, greater allowances should be made for this item of fixed cost. It now appears that too much of the burden is likely to be shifted to the investor, whereas, in all good conscience, a substantial portion should fall upon the consumer.

The interest problem comes down to this: Either Government development and ownership with its lower rates, or private development with interest rates fixed under the usual market conditions. As for taxes, the burden is shifted either to suppliers of the things the companies buy, to their customers, or to the investors; but when the average tax burden of the industry is 15% of the revenue, and higher in some instances, it becomes an extremely important matter. Unquestionably, it must go into costs, but the important question is whether it is to be recovered from the consumer or from the investor. If an attempt is made to recover so much from the investor that the rate of his return is lowered, he will transfer his attention to other investments and the capital needed for development will not be forthcoming.

The problem of joint costs is insoluble, and it is the same as that faced by the Australian sheep raiser who sells mutton, pelts for ladies' coats, and wool.

Incidentally, the comparisons made with foreign countries are of little validity in the present case. The social and economic milieu abroad, particularly, and the legal philosophy are so different that little is gained from their experience in solving problems in the United States.

If, as some believe, the nation is on the threshold of an era of decentralization, any prophecy of the social and economic consequences is futile; but it does suggest this: It would seem that proposals to establish, under Government ownership, various large units scattered throughout the nation as "yardsticks" is untimely, because it appears to commit the Government to a wasteful economic venture at the "tail end" or closing phase of engineering development. Certainly, if decentralization is to be the significant operating practice in the future, the Government should hesitate in the establishment of "seven little sisters" to the TVA.

Regulation in itself is an adequate instrument for social or economic problems. In fact, it is the best instrument possible, if used properly. With equal facility, one can either shave himself or cut his throat with a razor; it all depends upon the way one uses the instrument. Even in those cases where qualified economists, or capable engineers, sat on commisions, they found themselves confronted with political problems rather than ethical or economic ones. Many serious controversies have grown out of the abuse, and not the use, of regulation.

R. L. Thomas,[50] Esq. (by letter).[50a]—It is significant to the writer that Mr. Scharff, in referring to a 25-yr old report[21] on depreciation, selects as one statement with which there is still general agreement the comment that "perhaps there is no single subject in connection with valuation that has caused more trouble than depreciation." There seems to be something inherently controversial in the subject—something that causes blood pressures as well as voices to be raised. There is no subject on which it is more difficult to keep discussion within reasonable bounds—no subject so surrounded and obscured by "fog and smoke." Using another metaphor, the Hon. Clyde L. Seavey, Acting Chairman of the Federal Power Commission, in an address before the American Bar Association in August, 1936,[51] aptly referred to the "inferno of depreciation."

Engineers, more than any other class, should be striving to cut through the "haze"—to find a way out of the "inferno." They should bend their efforts toward simplifying rate-making procedure in the public utility field, provided that such simplification does no injustice to either the rate-payer or the utility.

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[50] Project Engr., Safe Harbor Water Power Corporation, Baltimore, Md.

[50a] Received by the Secretary July 14, 1938.

[21] *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), Chapter VI, p. 1448.

[51] *Public Utilities Fortnightly*, October 8, 1936, p. 546.

Mr. Scharff has done a real service in emphasizing the need for consistency between determination of depreciation for valuation and rate-base purposes and annual depreciation expense. His proposal that the actual depreciation be determined annually by mathematical formulas, with periodic checks by field inspection and engineering investigation, is theoretically desirable and is a logical method if it is necessary to use a depreciated rate base. Its practical application, however, is another matter. Even in the case of the simplest of the causes of depreciation—physical deterioration—the writer regretfully doubts that it will be possible to obtain general agreement on a series of "objective measurements" which will be recognized as measuring the extent of physical deterioration. With respect to the other causes, it seems even more likely that determination of the extent of depreciation will remain a matter of judgment. As a matter of fact, there are many cases in which it is impracticable to distinguish between several of the causes.

Now, there are undoubtedly many cases where there is no other "way out," and it is necessary to make the best possible determination of depreciation by a combination of "objective measurements," investigation, and judgment. A better way out of the "fog," when it is feasible, is the use of an undepreciated rate base. The complete sentence in Mr. Seavey's address[51] is: "And, in order to escape as far as possible from the inferno of depreciation, the California Commission used an undepreciated rate base with a correlative adjustment in expense."

Undoubtedly, the "correlative adjustment in expense" refers to the sinking-fund method of depreciation accounting which has been used in California for a long time. Under this method the total annual accretion to the depreciation reserve consists of two components—one the annuity which is chargeable to the rate-payers as an operating expense, and the other the interest on the reserve, which is not chargeable to the rate-payers, but is a deduction from income. It is not necessary to know the "actual" depreciation. The annuity, as Mr. Scharff states in his reference to special cases, equalizes the annual depreciation expense over a long period of years.

Five years of intimate knowledge of the use of the method at a large hydro-electric project has convinced the writer that the statement often made, that the method is complicated, is entirely unfounded. As compared with an annual or even a periodic determination of actual depreciation it is extremely simple.

Another method of using an undepreciated rate base is to determine the depreciation expense charged to the rate-payers by some simple and more or less arbitrary, but fair, method and credit the rate-payers with interest on the accumulated depreciation reserve at an arbitrary interest rate. This is the method used in the District of Columbia under the Consent Decree of the Supreme Court of the District of Columbia.

An objection sometimes made to the undepreciated rate base is that it is contrary to "accepted law." This statement was made recently by a well-known engineer before a Public Service Commission; yet it is common practice for electric utilities, including the company with which the aforementioned engineer is connected, to establish, among themselves and with large customers, annual payments for the use of facilities (as, for example, in interchange) by applying a fixed percentage to a constant sum of money. The rate base and the payment go on year after year without deduction for depreciation. Without attempting to argue the legal questions, the writer believes that if the method is equitable it will be found to be legal "in the long run."

THEODORE B. PARKER,[52] M. AM. SOC. C. E. (by letter).[52a]—In his paper on "Cost of Hydro-Generated Energy," Professor Barrows has developed a very effective method of analyzing such costs, and has tabulated cost data relative to a great variety of representative plants. Hydro-electric developments differ so widely in their general design, and must conform so frequently to special requirements, that it has been considered a difficult task to secure a common basis for comparison of costs. It is interesting, therefore, to find that Professor Barrows has managed to reduce a considerable number of variables to what appears to be a common denominator.

His method appears admirably adapted for making general comparisons of different plants. For a detailed study of a given power system it is naturally more accurate to use the actual characteristics of the system and its parts; that is, to determine the comparative costs of two alternative schemes for supplying equivalent generating capacity, the most reliable method is to compute the total system cost under each plan. "Short-cuts" or approximations are frequently misleading.

One of Professor Barrows' conclusions is that, "although, in general, the cost per kilowatt-hour tends to lessen as head increases and to lessen with the size of the plant, the effect of capacity factor is more predominant" (see heading, "Cost of Water-Power Output: Effect of Head, Capacity, and Capacity Factor upon Cost"). This statement can scarcely be disputed. On the other hand, the economy of high-head plants is sometimes more apparent than real. Plants with heads greater than 400 ft (with the exception of Boulder Dam and similar recent developments) generally require the use of long conduits. These conduits are frequently wood pipes or flumes which depreciate rapidly and require considerable maintenance. Where more permanent materials are used, there is still a greater proportional annual expense than for plants with the power-house directly at the dam. This tends to render incorrect a comparison of such plants based on the same fixed charges.

At a high-head plant the waterway frequently represents a large proportion of the investment. Its capacity must equal the capacity of the plant, with only minor allowance for load factor. Increment cost at such a plant, therefore, is high, and may affect the lower cost of generating equipment. For certain purposes it may be found more economical to secure additional peak capacity at the low-head plants of a system, for which the energy cost may be high but the capacity increment cost low.

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[52] Chf. Engr., TVA, Knoxville, Tenn.

[52a] Received by the Secretary July 19, 1938.

In Table 6 Professor Barrows includes certain high-head plants numbered 31 to 39, inclusive. These are comparatively small developments in Utah which were constructed prior to 1912 by various local interests, and, later, taken over by the Utah Power and Light Company. They were built at a time when the general cost level was lower than at present. The waterways are mostly of wood, and have required extensive maintenance and replacement which are apparently not reflected in the annual costs given in Table 6. Cost data from this group, therefore, are not a very reliable basis for appraising present-day, large-scale developments.

A study of Table 6 and the supporting text is most interesting and serves again to emphasize the fact that where exceptional economy is secured the cause is frequently some special condition rather than any general relation of head and flow. For example, the Chelan Project (Plant No. 43) owes its value largely to an opportunity to raise the level of a large lake by means of a relatively short dam at its outlet.

The effect of capacity variation is shown clearly in the case of the Rock Island Development (Plant No. 49), on the Columbia River. For the four present units the total cost is given as $14 555 000 and the unit energy cost as 0.40 cent per kw-hr. Since six additional units can be added at a cost not much exceeding $1 000 000 each, it is apparent that this will greatly reduce the cost per kilowatt-hour. Owing to the unusually well sustained flow there would be a large increase in total energy, but naturally not in proportion to installed capacity.

Table 6 shows a considerable number of reasonably large modern projects which develop energy at what might be regarded as high unit cost. This, it is assumed, must be justified by favorable load factors, high cost of power from other sources, or other local conditions. In general, it has been the writer's experience that outside of special conditions, such as existing lakes and unusually favorable topography, the most effective factor in securing economy is a high ratio between low flow and length of dam. Such a condition allows the most effective use of the structures built, and the greatest return on the investment.

In recent years, low-cost hydro-energy production has been obtained at developments where there are large, well-sustained stream flows, and where the power houses are directly at the dams. This is a most important factor in the very low costs obtainable at the large Federal developments on the Columbia River. It is a smaller, but still an important, factor in the relatively low costs at the plants on the Susquehanna River where the flow is large but not so well sustained. Many of the successful Canadian developments are situated on rivers having a large volume, with flow sustained by natural lakes on their head-waters.

Where long conduits are used there is a definite limit to the economy possible, except where a large drop can be obtained in a relatively short distance. This sometimes occurs where there is an opportunity to tunnel through a divide or to effect some similar short cut. To secure low cost per kilowatt-hour with long conduits, it is necessary to have a high capacity factor—that is, a high rate of use of the facilities provided.

In a given power system it is frequently desirable to determine where the cheapest generating capacity can be obtained; that is, to determine which plant can best be over-developed for use on the peak of the load. Other conditions being favorable, the lowest incremental cost is frequently found at a plant with moderately high head, but with the power house directly at the dam.

The most vital consideration affecting hydro-electric costs is the rate at which fixed charges are computed. Professor Barrows has taken fixed charges on the Federal hydro-electric projects at 6.5%, whereas the others are on a 10% basis. Of the fixed charges the largest factor is the interest rate, which varies with economic conditions and with the agencies involved. During recent years considerable Government financing has been effected for less than 3 per cent.

Depreciation (that is, purely physical depreciation) varies greatly with the type of development. Where most of the investment is in structures and equipment of limited life, the total average depreciation may exceed the 2% maximum rate given by Professor Barrows. On the other hand, there are certain types of development in which a large part of the investment is in items having practically no physical depreciation. An example of such items is the investment in reservoir facilities, consisting of cost of land, clearing operations, reconstruction of railroads and highways, and back-water protection.

In addition to the tabulation of cost data for fifty-seven hydro-electric developments, a summary is given of estimated costs for several of the larger Federal projects recently completed or under construction. Included among these is a summary of estimated costs for eleven existing or proposed major projects of the Tennessee Valley Authority.

Unit energy costs are given for two conditions: (1) Those based on power development cost alone, given as $135 000 000; and (2) those based on total cost, given as $520 000 000. Fixed charges are taken in each case at 6.5 per cent. The resulting unit costs are given as 0.183 cent and 0.60 cent per kw-hr, respectively. It is thus inferred that the cost of energy to the TVA lies between these limits.

The actual cost of energy to the TVA depends upon a number of factors, not all of which can be discussed adequately herein. It seems appropriate, however, to emphasize several factors. The actual interest rates paid by the Government during the TVA construction period have been very low. These rates are a matter of official record and can be verified easily. The physical depreciation of the TVA projects as a whole will be extremely low. More than one-third of the cost is in the reservoirs, with negligible depreciation, and much of the remainder is invested in massive structures of concrete and earth. The estimated annual depreciation of the dams, reservoirs, and generating stations (excluding transmission lines) will be less than 1% of the total cost.

Among the eleven projects listed as costing $520 000 000 is the Gilbertsville Project, which, if built, would be utilized mainly for flood control. Its estimated cost is more than \$110 000 000, which provides for only a moderate generating capacity. This can scarcely be classed primarily as a power development, and no theory of allocation would justify charging any considerable part of the total cost of this development to power. With Gilbertsville excluded, it is apparent that the upper limit of 0.60 cent per kw-hr would be reduced considerably even on the basis of 6.5% fixed charges. The effect of reducing the rate of fixed charges is obvious.

When final costs are available and definite allocations have been made, it should prove very interesting to analyze the TVA and other recent Government developments, and to compare them with the projects listed in Table 6. Meanwhile, the writer wishes to congratulate Professor Barrows on having developed a method which makes such comparisons practicable.

PHILIP SPORN,[53] M. AM. SOC. C. E. (by letter).[53a]—The question of true production economics is one to which engineers must pay more and more attention if more goods are to be available for distribution. Although the production of electric power and electric service has been developed to an unusual degree on sound technical and economic lines, the subject will bear further study and analysis. There have been periods in the past when the power business was growing at such a rate that almost the entire effort had to be centered upon the task of meeting demands. In such periods some mistakes—particularly economic mistakes—may have been made, but these did not show up and their cost was quickly absorbed in the growth of the business; but, as business approaches technical and economic maturity, the cost of such even rare mistakes is more apparent and it becomes necessary to scrutinize more closely the economic merits of what is done in the way of building new capacity. Although Mr. Whitman's paper traverses familiar ground, it may well be that the very repetition of these principles will "bring home," not only to engineers but to others who are or who may become associated with, or who are in contact with, the power business, the need for careful economic analysis in any power situation.

This subject of the cost of combined energy generation is one that requires, above all, careful analysis. So much has been said and written on the subject of steam *versus* hydro energy (with prejudice substituted for facts and figures), that fundamentals are being overlooked in the desire to "make out a good case" for whichever side its author has chosen.

In reviewing the elements entering into the cost of energy as enumerated by Mr. Whitman, the writer is impressed by the fact that not enough emphasis has been placed upon cost of transmission and its effect on the cost of the delivered energy. After all, that, and not the bus-bar cost at the generating station, is the item of greatest interest. Those who have studied the problem closely and thoroughly understand the economics of power supply have been, for the most part, unable to find any sound economical basis for long-distance transmission of power.[54] In general, energy is most economical when generated nearest the load. Beyond a certain distance, transmission definitely is not an economical proposition; and, only too often, hydro developments require some long transmission to reach the load. Long-distance transmission of electric energy had its early development in areas where fuel was scarce and costly and there was available hydro power which could be developed cheaply. The early days of the electric power development in California are a case in point. However, even there, this situation has changed with the discovery of oil and gas supplies near the load. The writer feels that this question of transmission cost is very definitely a limiting factor in the development of large hydro-electric projects. Therefore, when a question arises as to whether a hydro-plant or a steam plant should be built to meet the growth of load, one should be very careful not to overlook the cost of transmission.

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[53a] Received by the Secretary July 21, 1938.

[54] *Proceedings*, Am. Soc. C. E., December, 1937, p. 1929.

In illustrating the economic principles of selecting the source of power, Mr. Whitman chooses the well-known situation at the Conowingo Plant, on the Susquehanna River. He could not have picked a better one. Everything was in favor of fitting that plant into the load curve economically: (1) Short transmission distance (only about 60 miles) to a large load center; (2) access to a large metropolitan load, rapidly growing, in which fuel costs were relatively high; and (3) an existing steam system in which there was sufficient steam-plant capacity to firm the larger part of the Conowingo capacity. One wonders how the Conowingo Plant, if proposed to-day, would appear in the light of the latest developments in steam-plant design. Mr. Whitman states that a new steam plant was estimated at that time to cost \$135 per kw. The most pessimistic engineer, to-day, would expect to reduce this cost by at least \$25 per kw, and if he were in an optimistic mood, by as much as \$45 per kw. Mr. Whitman does not state what efficiency was estimated for the plant, but certainly the efficiencies (that is, the fuel consumption) estimated in that day for the new plant were not as good as those that would be estimated to-day with all the new knowledge of high-pressure and high-temperature operations. In 1925, engineers were thinking of plants with a fuel consumption of 13 500 Btu per kw-hr. To-day, 10 000 Btu per kw-hr is "within sight."

This leads to another thought. The common impression is that hydro-plants never wear out and, once built, are useful for all time. Granted that hydro-plants and equipment show the effects of wear and tear slowly, one still must not overlook the influence of obsolescence on these plants, an effect that has been accelerated in the ten years since 1928 by the very great improvements in generation of energy by steam. All those conversant with the facts admit freely that the great improvements in the art of generation of energy by steam have made the development of new hydro-plants less attractive; but it is often forgotten that the same advance in steam generation has made present hydro-plants less of a bargain than they appeared to be when built.

Of course, the plants are built, the money has been spent, and they will continue to be operated until their cost becomes prohibitive in relation to some new future developments. The writer wishes to emphasize this point on obsolescence to show that it may be a much more important factor in the determination of the economic value of hydro-plants than is generally assumed.

However, he does not wish to be misunderstood in regard to hydro-electric power development. He has no prejudice against it; nothing but a desire to examine the facts clearly. The American Gas and Electric Company has in operation about 125 000 kw of hydro-electric capacity fitted into what is basically a steam system, with about 850 000 kw of installed steam capacity.

The writer would like, also, to mention the subject of Federal hydro-projects which Mr. Whitman covers in the closing paragraphs of his paper. In concentrating on engineering calculations, one should not lose sight of some of the social implications in the development of large hydro-plants by the Federal Government. As indicated in the principles and data presented by Mr. Whitman, hydro developments are generally feasible, economically, only when they are made complementary to steam developments. To make them directly competitive, some form of subsidy is necessary and, if this subsidy is made by the National Government, the national economy cannot help but suffer. Moreover, a subsidy that brings this about definitely tends to reduce the national pay-roll.

Although it is well known to most engineers connected with the development of power, it is not generally appreciated that in a hydro-electric project, the principal cost of the energy is the fixed charges. Very little will go into operating expenses in the form of labor and materials.[55] In a steam-generating plant, on the contrary, fully 50% of the total cost of power goes into continuously recurring expenses for material, fuel, and labor. It has been estimated that for every billion kilowatt-hours generated each year by hydro, there is less paid out for labor and material by the approximate sum of $2 300 000. This amount is obviously reflected in further reduction in purchasing power and pay-rolls through the well-known downward spiral, so that it is entirely probable that this process results in as much as $10 000 000 to $15 000 000 of annual decrease in the national pay-roll for every billion kilowatt-hours of hydro-electric energy generated. Taking, as a fair assumption, the maximum possible hydro-energy that might be developed by subsidy along these lines at 20 000 000 000 kw-hr per yr, it can be seen that the production of such an amount of energy would mean a reduction in the annual pay-roll of between $200 000 000 and $300 000 000. This most certainly is not a negligible item in the national economy; particularly, since it will result in more expensive power to every one and the only beneficiaries of the development along these lines will be those who will be provided with additional means and outlets for investments of their capital in tax-exempt Government securities.

L. N. McClellan,[56] Esq. (by letter).[56a]—The cost of power developed in connection with the large multi-purpose projects that have been undertaken in recent years by the Federal Government depends upon two important factors: (1) The extent to which the cost of these projects is allocated to purposes other than power, such as flood control, improvement of navigation, conservation of water for irrigation and domestic uses, etc.; and (2) the period of time required to absorb the full power output. These facts have been stated by Professor Barrows and Mr. Page.

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[55] *Proceedings*, Am. Soc. C. E., December, 1937, p. 1927.

[56] Chf. Elec. Engr., U. S. Bureau of Reclamation, Denver, Colo.

[56a] Received by the Secretary July 23, 1938.

Congress recognized the flood control benefits of the Boulder Canyon Project and allocated \$25 000 000 of the cost of the project to flood control to be repaid out of 62.5% of revenues in excess of those required to amortize the remainder of the cost of the project. The other 37.5% of the excess revenues is to be paid to the States of Arizona and Nevada in lieu of taxes. The Boulder Canyon Project Act provides that all funds advanced for the construction of the project are to be repaid in 50 yr with interest at 4% per annum. All Boulder Canyon power has been contracted for and the power contracts provide a definite date for each power contractor to start taking power and each contractor has a 3-yr period in which to absorb the full amount for which he has contracted. Therefore, this project is on a definite basis as to the allocation of cost and period required to market the power. The estimated cost of the Boulder Canyon Project, with full installation of generating equipment, is \$143 000 000, including interest during construction. The estimated annual cost of the Boulder Canyon Project is given in Table 16 (Item No. 1), assuming that amortization of the \$25 000 000 allocated to flood control is included in the rate base.

TABLE 16.—ESTIMATED ANNUAL COST OF BOULDER CANYON PROJECT

Item No.	Assuming that the amortization of the \$25 000 000 allocated to flood control is:	Interest rate, percentage (1)	Cost, in Thousands of Dollars				
			Interest and amortization (2)	Depreciation (3)	Operation and maintenance (4)	Payments to Arizona and Nevada* (5)	Total annual cost (6)
1	Included in the rate base.........	4.655	6 657	802	841	700	9 000
2	Not included in the rate base.....	4.263	5 030	880	841	700	7 451

* In lieu of taxes.

Revenue from stored waters will average about \$250 000 per yr, and that from secondary energy will average \$350 000 per yr, assuming that the present rate of 0.5 mill per kw-hr for secondary energy remains unchanged. Deducting these sums from the total annual cost leaves \$8 400 000 as the annual cost chargeable to firm energy. The average annual output of firm energy during the 50-yr amortization period will be 4 120 million kw-hr, and the average cost of firm energy will be 2.04 mills per kw-hr delivered at the high-voltage bus at Boulder Power Plant. The present rate for use of falling water for generation of firm energy is 1.63 mills per kw-hr, in addition to which the power contractors are required to repay the cost of the generating equipment and to bear the cost of the operation and maintenance of the power plant. At 55% load factor these additional costs amount to about 0.57 mill per kw-hr making the total cost of power under the present contracts about 2.2 mills per kw-hr.

There has been considerable agitation on the part of the contractors for Boulder power to have the interest rate reduced from 4% to 3.5%, which they contend would be more in line with the present rate of interest paid by the Federal Treasury on borrowed money, and to defer repayment of the $25 000 000 allocated to flood control until the remainder of the cost of the project has been repaid. These changes would require additional legislation and if they were made the annual costs would be reduced as indicated in Item No. 2, Table 16.

Deducting the revenues from stored water and secondary energy amounting to $600 000 per yr from the total annual cost of $7 451 000, would leave $6 851 000 as the cost of firm energy. On this basis the cost of firm energy would be 1.66 mills per kw-hr delivered at the high-voltage bus at Boulder Power Plant. This corresponds with 1.7 mills per kw-hr reported by Professor Barrows for the cost of Boulder Canyon energy. Reducing the rate of interest from 4% to 3.5% per annum and eliminating the $25 000 000 allocated to flood control from the power rate base would reduce the cost of firm energy delivered at the high-voltage bus at Boulder Power Plant about 18.6 per cent.

The Grand Coulee Project on the Columbia River in Eastern Washington is the largest of the multi-purpose Federal projects. It will make available almost twice as much firm energy as the Boulder Canyon Project and no part of this power has been contracted for to date (July, 1938). The project is a combination irrigation and power development, and power revenues are expected to repay a large part of the cost of the irrigation development as well as the entire cost of the power development.

The Grand Coulee Dam and Reservoir will serve both irrigation and power and, because of this dual purpose, it has been tentatively proposed to allocate one-half the cost of the dam and reservoir to irrigation and one-half to power. The total estimated cost of the project is $425 000 000. Of this sum, $282 000 000 is the cost of the irrigation development which is to be repaid in 40 yr without interest (conforming with the established plan of repayment of Federal Reclamation projects), and $143 000 000 is the cost of the power development which is to be repaid in 40 yr with interest at 3.5% per annum. It is estimated that the water users will be able to repay about $23 000 000 of the construction cost of the irrigation development in addition to the cost of operation and maintenance, leaving about $243 000 000 of the cost of the irrigation development to be repaid out of power revenues.

With the 5 000 000 acre-ft of active storage in the Columbia River Reservoir, the firm energy output of the Grand Coulee Power Plant will amount to 8 138 million kw-hr per yr. A considerable period of time will be required for the market within economic transmission distance to absorb this large amount of energy and for the purpose of determining the cost of power it is assumed that about 20 yr will be required to market all the firm energy. It is believed that this is conservative, and it is expected that the full output will be absorbed in a considerably shorter period. If so, the deficits in the early years and the cost of power will be reduced somewhat. In addition to the firm energy there will be large amounts of seasonal or secondary energy which it is proposed to use largely for pumping water for irrigating the lands of the Columbia Basin Project. It is assumed that the water users will pay 0.5 mill per kw-hr for secondary energy used for pumping purposes.

An analysis of the financial operation of the Grand Coulee Project is shown graphically in Fig. 34, which is based on the assumptions that the firm energy will be absorbed at the rate of 720 million kw-hr during the first year of operation, increasing at the rate of 360 million kw-hr per yr until the twenty-second year when the annual output will be 8 138 million kw-hr. The irrigation development will be 50 000 acres in the first year and will increase at the rate of 50 000 acres per yr until the total of 1 200 000 acres has been developed.

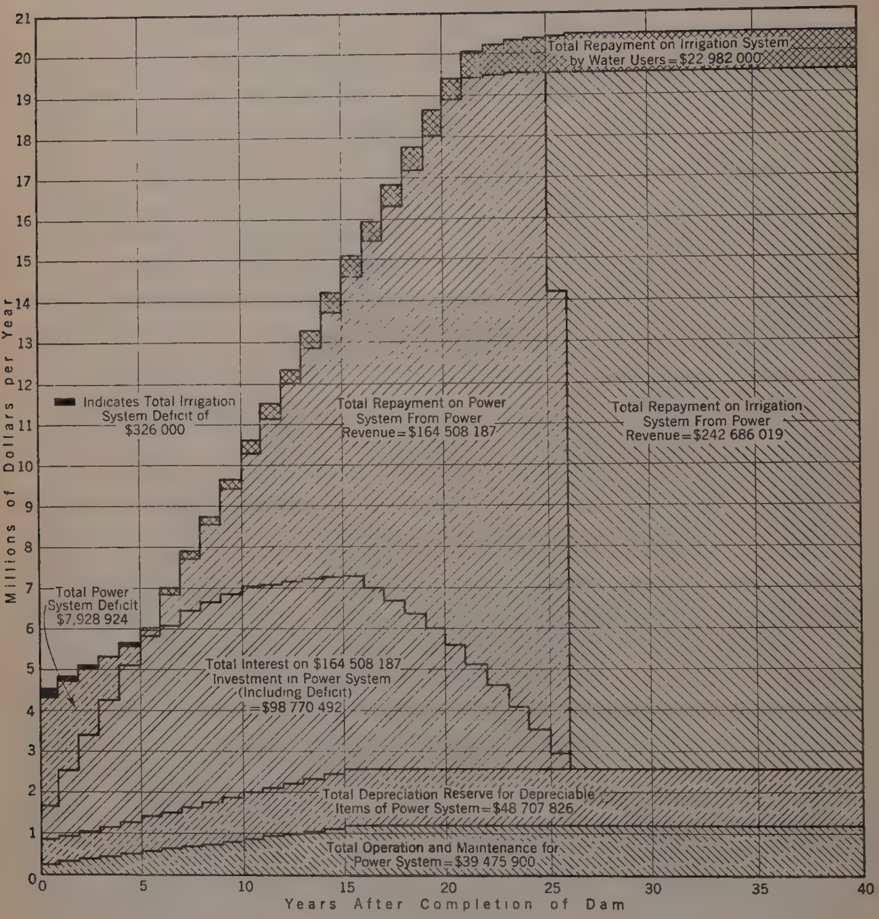

Fig. 34.—Power Development, Columbia Basin Project

Interest is computed at 3.5% per annum on the power system investment and no interest is charged against the irrigation system. Fig. 34 indicates that during the first five years of operation the revenues will be insufficient to meet the annual interest charges on the investment in the power development, and deficits amounting to about $8 000 000 will accumulate which will have to be paid off with interest in subsequent years, adding to the cost of power.

Interest payments will reach a maximum in the sixteenth year, after which they will reduce rapidly as the investment in the power development is amortized. The cost of the power development will be amortized in 26 yr and the cost allocated to irrigation will be repaid in about 41 yr. This study indicates that the cost of firm energy delivered at the high-voltage bus at Grand Coulee Power Plant will be 2.25 mills per kw-hr if repayment of the cost of the irrigation development, to the extent of about $243 000 000, is included in the power rate base.

Professor Barrows considered the cost of Grand Coulee power on the assumptions that the investment in the power development only is included in the power rate base and that the total energy available is disposed of; under these conditions he found that the cost of energy would be 1.5 mills per kw-hr based on the total output, including both firm and secondary energy.

The propriety of subsidizing irrigation development with revenues from power might be subject to question, but it must be borne in mind that the water users cannot, and should not, be expected to shoulder the entire cost of an irrigation development, such as the Columbia Basin Project. If the proposed cost of firm energy is competitive with the cost of energy from other sources in the Northwest and the power users are able to secure power from the Grand Coulee Dam as cheap or cheaper than it can be obtained from other sources they will not suffer by the proposed subsidy of the irrigation development. In fact, they will benefit from the general increase in the wealth and commerce of the Northwest as a result of the development of the 1 200 000 acres of the Columbia Basin Project. Considering the Columbia Basin Project as a whole it would seem entirely proper to apply power revenues to the repayment of the cost of part of the irrigation development in order to place the project on a sound financial basis and make it economically feasible.

JAMES C. BONBRIGHT,[57] ESQ. (by letter).[57a]—Although Mr. Scharff is thinking primarily in terms of proper "financial" accounting for depreciation, the writer wishes to add some comments in terms of the related but distinct problem of "cost" accounting. What a cost accountant properly treats as a cost is often something quite different from what a company auditor properly treats as one.

First, the writer wishes to discuss the question why any allowance for depreciation need be made in estimating, say, the average kilowatt-hour cost of generation by a given steam plant at 60% load factor. Presumably, the answer is that the economic life expectancy of the plant assets must be prophesied in order to determine the total number of units of energy against which the initial cost of constructing the plant must be charged. If a new turbine were required for each kilowatt-hour of energy to be produced, no allowance for depreciation would be needed. The entire cost of the turbine would then be charged directly to each kilowatt-hour, with the result that electricity would be prohibitive except for experimental purposes. On the other hand,

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[57] Prof. of Finance, Columbia Univ., New York, N. Y.

[57a] Received by the Secretary July 24, 1938.

if the turbine and its parts were indestructible, no depreciation allowance would be required, and the entire fixed charges on that asset would be limited to interest and taxes. In fact, however, neither of these extremes will be realized. The turbine will have, say, twenty years of useful life, and then it will be retired. Therefore, in cost calculations, allowance must be made for this 20-yr life expectancy as the only means of estimating what total volume of energy should assume the burden of the cost of buying and installing the turbine.

The writer is well aware of the hazardous nature of forecasts as to the future retirement dates of turbines and other assets used in the production and supply of electricity. The difficulties are so great that a plausible, although not convincing, case can be made for a refusal to make the forecasts for purposes of standard, "financial" accounting. In "cost" accounting, however (which is the only type of accounting that concerns the subject of this Symposium), there is no escape from the necessity of forecasting the future; and an assertion that life expectancy cannot be estimated is tantamount to an assertion that the cost of energy generation is indeterminate.

To restate the problem: In cost analysis, the problem of allowing for depreciation is the problem of cost pro-ration. It is the problem of tying up a given capital outlay—say $1 000 000 for a steam turbine built in 1937—with that estimated volume of energy which can economically be produced by the use of the asset for which the outlay was made. Strictly, depreciation is not a cost, because the entire cost of the turbine was in fact incurred in 1937, when it was paid for by the purchasing company, and no accounting mechanism by which this cost is gradually amortized over a period of years can alter the historical fact that the full cost was actually incurred in that year. Depreciation is a factor, however, that should determine what units of output should be held responsible for the total outlay and hence is a necessary factor in the calculation of average unit costs.

The foregoing statements are very general and, perhaps, very obvious; but the following is not so obvious and seems worthy of more attention than it has received. It concerns the allowance for depreciation due to prospective obsolescence in estimates of energy cost designed to show which of any alternative methods of producing energy is the most economical. Current practice in this respect is sometimes in error. Suppose that a manufacturer making large-scale use of electrical energy is in doubt as to whether to build a private steam plant or a private hydro-plant, since the alternative cost estimates come quite close together. The hydro-plant, say, would be scrapped in fifty years, because of silting and other elements of physical depreciation, but would not become obsolete before that date. The steam plant, with adequate replacement of wearing parts, would also have a 50-yr physical life but would become obsolete in twenty years.

Now ordinary practice would require that the cost of the steam plant be written off in twenty years, whereas the cost of the hydro-plant would be written off only in fifty years; and this mere difference in life expectancy might seem to turn the scales in favor of the hydro-plant because of its gentler liability to depreciation. Water power might appear to cost 5 mills; steam power might be shown to cost 5.5 mills. Therefore, the data would seem to say: "Let the manufacturer build the hydro-plant rather than the steam plant."

Is there not something wrong with this picture? Why should the choice lie with the hydro-plant simply because water power has already come close to realizing its maximum efficiency whereas a much better steam plant may be available in the near future? To be sure, if the manufacturer can afford to postpone building any power plant, his expectation that a better steam plant will be available within a few years may possibly justify delay in building; but if he must build some plant now, why should he decide against the steam plant merely because the hydro-plant, once built, will take a longer time to become obsolete than the steam plant?

This is a problem that the writer has not yet solved to his own satisfaction; but he suggests that, in calculating alternative costs, the engineer should not burden his putative steam-plant costs with any allowance for obsoleteness antedating the retirement date of the hydro-plant, unless he also makes a specific allowance for the lower operating expenses that the manufacturer will be able to enjoy during the remaining life of the hydro-plant if he now builds a steam plant, retires it in twenty years, and then builds a more efficient new steam plant. In practice, however, an engineer can scarcely forecast the operating expenses of a steam plant to be designed twenty years in advance of the present time. His ignorance, therefore, may force him to assume that the annual expenses of steam generation, during the longer life of a hydro-plant, will remain what they are to-day; but if he makes this assumption, he should not burden his steam-plant cost with an allowance for a premature obsoleteness which, should it materialize, would put the owner in a position to take advantage of the very improvement that will have rendered his present steam plant obsolete.

To state the point in another way: In choosing between the construction of a high-cost hydro-plant with a probable useful life of fifty years, and the construction of a lower-cost steam plant with a probable useful life of twenty years, the choice cannot be based intelligently on a mere comparison between the average cost of the hydro energy for the longer period and the average cost of the steam energy for the shorter period. Instead, the comparison should be between the cost of the hydro energy over the 50-yr estimated life of the hydro-plant, and some average cost of energy for the same period produced, first by a present-day type of steam plant and then by whatever type of new plant will become economical when this steam plant is retired. However, if it is impossible, for practical reasons, to estimate what it will cost to produce energy by a new plant designed twenty years from the present date, and if, therefore, the cost analyst must assume the continuance of the present annual steam operating expenses for the entire 50-yr period, he should not contradict this assumption by calculating an annual allowance for steam-plant depreciation that assumes a state of obsoleteness within twenty years. If the steam plant should indeed become obsolete in twenty years, that very fact would mean that the company which built this plant could then reduce its energy costs by installing a new plant. No similar opportunity to take advantage of rapid improvements in technique will apply if the company decides to build the hydro-plant, because the costs of hydro generation are largely fixed costs that are unavoidably "sunk" as soon as the plant is constructed.

The writer is not at all sure that he is right in this tentative solution of the foregoing problem; but the problem is a real one and has not yet received adequate study by specialists in cost accounting. In general, the problem arises because obsolescence, unlike physical deterioration, is only the adverse side of a good situation—of an opportunity to reduce costs in the future. The question arises as to whether cost accounting should not take direct cognizance of this good side instead of giving its sole attention to the bad side. To the important questions which Mr. Scharff has suggested for study, the writer would add this further question as offering an almost virgin field for engineers interested in private and public cost accounting.

It is pertinent to raise a question as to the definition of that multi-significant word, "depreciation." For practical reasons, the writer is inclined to favor the recognition of a number of alternative definitions, each of which might be characterized by some distinguishing adjective or adjunct. Although the restriction of the word to just one meaning would have obvious advantages, the difficulty of finding words for other accepted meanings is formidable. Engineers would probably find it difficult, if not impossible, to induce accountants, appraisers, and Courts to limit the use of the term to one single sense.

Mr. Scharff's proposal to use the term as an expression for the combined effect of a number of specified "causes" or phenomena (wear and tear, obsolescence, etc.), raises the question: Effect on what? In ordinary appraisal and accounting usage, depreciation is treated as something measurable in terms of dollars. Consequently, it is usually taken to denote a decline in value due to certain causes. Must one not say, therefore, "combined effect on value"? And must one not, then recognize that, since "value" itself is a word of many meanings, "depreciation" is indeterminate without reference to the type of "value" to which it is applied?

Finally, there is a distinction between depreciation as an economic value concept and deterioration as a physical concept. Referring, for example, to wooden poles, Mr. Scharff suggests (see heading "Physical Deterioration") that "deterioration through fiber rot in such a case may reasonably be measured in terms of decline in compression and bending strength at standard fiber stresses." Yet, the decline in the value of the pole resulting from the process of rotting (depreciation) is by no means closely correlated with the decline in compression or bending strength. Similarly, an ornamental candle, half used up (or even burned half-way down to the top of its candlestick) is worth less than one-half its value new, because of the serious impairment in its ornamental qualities. The writer suspects that appraisal engineers, in their testimony in valuation cases, are sometimes guilty of making unwarranted transfers from engineering concepts such as that of "per cent condition" to economic concepts such as that of "per cent of value new." Precisely what is meant by "per cent condition"? The writer has never found an appraisal expert who could answer that question with even a decent approach to scientific precision.

R. Robinson Rowe,[58] M. Am. Soc. C. E. (by letter).[58a]—Under the heading, "Undeveloped Hydro-Electric Power," Professor Barrows has tabulated the "horse-power, potential water power (available 90% of time)" for the ten leading States. The writer has been greatly impressed by the quantity of cheap water power available in Southeastern Alaska (sometimes called the "Panhandle"). Although not proposing Statehood for all or any part of Alaska at this time, it is suggested that the southeastern part has an area comparable to that of many States, a population density not unlike that of Nevada, a climate less rigorous than that of many States, and social, political, and economic qualities equivalent to those of many States at the time of admission.

Comparison with power resources of leading States seems in order and the comparison is more striking if expressed in terms of power density. This density has been computed on the basis of political boundaries, but, if data were available, and density were computed to water-shed lines, the comparison would be still more impressive. Less than 5 000 hp of that surveyed is available from water that has crossed the political boundary of the area. The data in Table 17 have been compiled from Professor Barrows' table, a survey[59] of Southeastern Alaska made for the U. S. Forest Service by the writer, and data from publications of the U. S. Geological Survey.

TABLE 17.—Water Power Density

Item No. (1)	State, or region (2)	Horse-power, potential water power* (3)	Area, in square miles (4)	Horse-power per square mile (5)	Item No. (1)	State, or region (2)	Horse-power, potential water power* (3)	Area, in square miles (4)	Horse-power per square mile (5)
1	Washington	8 800 000	69 127	127	7	Tennessee	1 300 000	42 022	31
2	New York	4 300 000	49 204	87	8	California	4 600 000	158 997	29
3	Connecticut	330 000	4 845	66	9	Southeastern Alaska	790 000	35 527	22
4	Oregon	4 400 000	96 699	45	10	New Hampshire	200 000	9 031	21
5	Arizona	3 700 000	113 956	33	11	Utah	1 500 000	84 990	18
6	Idaho	2 700 000	83 888	32	12	Alabama	900 000	51 998	17

* Available 90% of time.

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[58] Assoc. Bridge Engr., Dept. of Public Works, Sacramento, Calif.

[58a] Received by the Secretary July 30, 1938.

[59] Report to be published shortly by the U. S. Forest Service and the Federal Power Commission.

Summarized, the survey of Southeastern Alaska power yielded the following totals:

Hydro-Power Potential:		
Average yield, in horse-power		1 007 590
Primary yield, in horse-power		790 470
Hydro-Power Developed:		
Average yield, in horse-power	36 064	
Abandoned yield, in horse-power	−8 190	
Net, active yield, in horse-power		27 874
Installed:		
Turbines, horse-power		40 676
Generators, kilovolt-amperes		25 811

Many of these powers can be developed for less than $50 per hp, as the physiography is favorable to low cost and most power-house sites are at protected tidewater. Storage reservoirs can be developed from "hanging lakes," in elevated valleys left by receding glaciers close to tidewater. Heads average 400 to 500 ft, with a maximum of 2 400 ft. Conduit lengths generally approximate five times the head and mountain spurs will usually permit about 70% of the conduit to be a low-pressure or a moderate-pressure line. Precipitation is high, sea-level annual means ranging from 27 in. at Skagway to 180 in. at Little Port Walter, and annual means of precipitation and run-off show less departure from norms than for the Pacific Coast States.

V. B. LIBBEY,[59] ASSOC. M. AM. SOC. C. E. (by letter).[59a]—The writer doubts whether a technician entering the forum constituted by the *Proceedings* of the Society to discuss "depreciation" as applied to units of public utility property is free to write his own definition of the word, and he believes it imperative that this point should be brought out in the discussion of Mr. Scharff's paper.

The larger phase, of course is whether or not the concept presented is necessary or usable in the philosophy or technique of rate determination or other problems in engineering economics. This discussion will be limited to a consideration of the first proposition, with the larger question entering only incidentally.

"Depreciation" (the word must remain in quotation marks until a definition is adduced) of units of a public utility property is a most important factor in the determination of rates (especially as the rates are determined by administrative boards) in that it appears as an element of "value" and again as an element of expense. As stated by the Special Committee on Valuation and quoted by Mr. Scharff:[21] "Perhaps there is no single subject * * * that has caused more trouble than depreciation." Continuing (in the following sentence not quoted by Mr. Scharff) the Committee stated: "This has been due to various causes, perhaps not the least of which has been confusion in the use of the term."

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[59] Supt., Hawaiian Contracting Co., Ltd., Honolulu, Hawaii.

[59a] Received by the Secretary July 30, 1938.

[21] *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), Chapter VI, p. 1448.

The deliberate introduction of confusion into rate proceedings in the forum constituted by public utility commissions acting as judicial bodies and subject to review by the Courts seems to be an accepted—if not highly approved—technique followed, in the legal approach to rate-making by counsel employed as advocates of low rates or high rates. In such instances, the proceeding partakes of a trial, and the facts are to be determined on the weight of the polar evidence presented by advocates approaching the matter from the two extremes, and employed and paid in accordance with their ability to influence the decision.

On the contrary, in the engineer's forum the object must be to eliminate confusion. The engineer seeks to determine the operations to which the concept is subject and, if possible, fix the concept by fixing the operations. In this respect the engineer must maintain an attitude toward depreciation much the same as that which he maintains toward length, velocity, and other phenomena with which he deals.

Even in proceedings before commissions confusion as to the use of the term has been greatly reduced, and among economists, engineers, accountants, etc., the concept of the word has been crystallized to mean loss of value arising from causes that bring about ultimate retirement.

In a remarkable manner the Special Committee on Valuation, beginning its work in 1911 and reporting in 1916, viewed "depreciation" in exactly this manner. Only in 1909 had the United States Supreme Court, with a background of legal philosophy as to property and value and its body of laboriously built precedent, recognized "depreciation" as a loss of value. Even so the Court had left it as a metaphysical concept not clearly related to its causes. The Committee of engineers trained in an objective approach, went directly to the causes. Accustomed to the use of mathematics as a tool for the transposition of an unusable set of "knowns" to a usable "expression," the Committee set forth a method of arriving at "depreciation" which satisfied the requirements of fairness and convenience that were involved in the concept and were believed to conform to law. If "remarkable" carries a suggestion of surprise, then "noteworthy" is a better word, because the engineers acted only in accordance with the customary engineering method of approach.

The following is a definition of "depreciation" that may be obtained from the text of the report of this Special Committee by a grouping and re-arrangement of words and phrases, with only slight modification:

> "Depreciation," as applied to depreciable electric plant, means the loss in service value not restored by current maintenance, incurred in connection with the consumption or prospective retirement of electric plant in the course of service from causes which are known to be in current operation and against which the utility is not protected by insurance. Among the causes to be given consideration are wear and tear, decay, action of the elements, inadequacy, obsolescence, changes in the art, changes in demand and requirements of public authorities.

The foregoing does not embrace the entire concept of the Committee but, in so far as it goes, it is in strict accord with it. It is the definition of depreciation formulated by the Federal Power Commission[60] in its new classification of accounts mentioned under the heading, "Public Utilities," in Mr. Scharff's paper.

This is also (except that "utility" is substituted for "electric" as the adjective qualifying "plant") the definition appearing in the Uniform System of Accounts recommended for adoption by the National Association of Railroad and Utilities Commissioners[61] and it is already quite generally adopted by State regulatory commissions. It is almost identically the definition of depreciation adopted by the Interstate Commerce Commission after extensive hearings on the matter of Depreciation Charges of Telephone Companies and Steam Railroad Companies.[62] It is almost, word for word, the definition appearing in Uniform System of Accounts for Telephone Companies prescribed by the Federal Communications Commission.[63]

To the layman, at least, it is a very accurate definition of depreciation as viewed by the Bureau of Internal Revenue in enforcing the provisions of the Income Tax Law.[64]

It is thus seen to be the definition of "the various administrative bureaus and commissions, necessarily called * * * into existence by the increasing complexities of our modern business and political affairs."[65]

As was not the case in 1916, when the report of the Special Committee on Valuation was rendered, this definition is now clearly accepted by the Supreme Court. In the Lindheimer *v.* Illinois Bell Telephone Company Case, Chief Justice Hughes, speaking for the Court says:[66]

> "Broadly speaking, depreciation is the loss, not restored by current maintenance, which is due to all the factors causing the ultimate retirement of the property. These factors embrace wear and tear, decay, inadequacy and obsolescence."

The engineers, the administrative bodies, and the Courts have thus tied depreciation to the essential observable operation—ultimate retirement. This is the connecting link between the abstraction, loss of value, and its causes without which there would be a "missing link." It is the integrating medium.

The definition does not fix the operation by which depreciation is to be measured, but it leaves the way open. At this stage, with past practices as to amortization so diversified it is probably expedient, reasonable, just, and equitable that this should be so. The engineer and other technicians who have a working acquaintance with mathematics can set up a rate of return and anticipated lives in years that can be agreed upon or determined by the weight of the evidence, and can develop formulas for present worth of future returns (value) that will permit arriving at loss of value. This, in the mind of the writer, will be "true depreciation." The annual charge to expense for depreciation so computed will be less than that under the "straight line" method of computation for expanding enterprises, such as power industries, and will permit lower rates that, in turn, will promote additional consumption.

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[60] "Uniform System of Accounts Prescribed for Public Utilities and Licensees Subject to the Provisions of the Federal Power Act," Federal Power Commission (Approved June 16, 1936), Definition 13, p. 5.

[61] "Uniform System of Accounts for Electric Utilities," National Assoc. of Railroad and Utilities Commrs. (Copyright, 1937), Definition 13, p. 3.

[62] "Depreciation Charges of Telephone Companies, No. 14700"; and "Depreciation Charges of Steam Railroad Companies, No. 15100"; Interstate Commerce Commission; decided July 28, 1931; 177 I. C. C., p. 422.

[63] "Uniform System of Accounts for Telephone Companies," Prescribed by the Federal Communications Commission in accordance with the provisions of the Communications Act of 1934; issue of June 19, 1935; Definition "J," p. 3.

[64] Revenue Act of 1934, Sections 11, 21, 22, and 23, Regulation 86, Article 23, Treasury Dept., U. S. Bureau of Internal Revenue.

[65] Mr. Justice Sutherland, Jones *v.* S. E. C. 298 U. S. 1 (1936).

[66] Public Utility Repts. (New Ser.), 292 U. S. 151 (1934).

Meanwhile, there are utilities which have followed a "retirement method" of accruing depreciation upon which it would work a great injustice to allow a return upon a "value" so computed. There are other utilities which have followed the "straight line method" of accruing depreciation. To compute rates on the higher value that would obtain and to include in expense the higher depreciation allowances that accure in the later life of units would impose an unfair and unreasonable burden on consumers.

As Mr. Scharff observes (see "Introduction"):

> "It is a sound financial principle that the value of a capital investment is equal to the present worth of the amortization of the investment plus the return upon the unamortized portion of the investment. This principle can be complied with, however, by the consistent application of any method of amortization whatever, * * *."

Equitably and reasonably the method chosen for amortization may be conceived of as determining loss of value and thus depreciation. This is the case if "anticipated returns" is the measure of value, and a "lack of relation to depreciation" becomes impossible.

Under the definition of depreciation presented herein and holding to the "present worth" concept of value, the rate-base of a utility may be fixed, whatever the method of amortization used in the past, and substantial justice will be done to owners and consumers. Acceptance of the definition and the "present worth" concept of value results in a workable plan for the inclusion of depreciation in rate determination or other problems in engineering economics that involve depreciation.

EDWIN D. DREYFUS,[67] ESQ. (by letter).[67a]—An important contribution to the problem dealing with depreciation is presented by Mr. Scharff who offers a novel plan for regularly checking up on the extent to which depreciation has actually progressed.

From a theoretical aspect, it is a simple matter to assume certain life probabilities and then make mathematical calculations for renewal or retirement accruals, more commonly called "depreciation." On such premises, a perfect balance can be obtained on a paper basis, as between the annual accrual rate and the accumulated reserves. The result of such calculations has the appearance of precision, and possibly the application of well informed judgment also may be implied accordingly. Extensive treatises have been published attempting to prove that accruals may be determined rather definitely. However, close investigation will reveal clearly that the depreciation problem is not quite so simple. As things happen to work out in Nature, there are always particular conditions or characteristics that Man must cope with and satisfy according to the practical necessities of the situation. The following will demonstrate this contention. A new business venture does not develop its full earning power at its inception; nor is it able to maintain earnings at required levels up to the time of final abandonment. Furthermore, it is not to be presumed that any business may be continued in perpetuity no matter how essential or vital it may be at the moment. Through the ingenuity of Man, the possibility of a substitute being developed is always in the offing.

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[67] Engr., Pittsburgh, Pa.

[67a] Received by the Secretary August 26, 1938.

The fact must always be remembered that through governmental regulation, the return is limited on the one hand, and that there is, on the other hand, no corresponding guaranty of the investment or protection against losses. By dealing too closely with various features of the regulatory problem, regulators, actuarians, accountants, engineers, and others might readily produce an undesired effect, causing real injury and thereby discouraging progress.

The estimating of allowances for annual accruals for depreciation is generally approached on theee much discussed bases: Straight-Line Method; Sinking Fund Method; and Retirement Method. There are some variations, but it is not important for the practical objective to include them in this discussion. The first two are based upon assumed life tables and the third, the Retirement Method, involves a careful gauging of the requirements from time to time—such as business experience in the past, progress in the arts, changes in governmental activity, and other influences would dictate.

In order that a better idea of the true depreciation problem may be had, the life of a business enterprise (and particularly one of a highly technical character such as that under consideration) is divided (see Fig. 35) into four stages: (1) Pioneering Period; (2) Accelerating Expansion; (3) Possible Arrested Growth; and, (4) Possible Serious Decline.

It is generally known that it requires time to place a new type of operation on a paying basis. A certain pioneering or developmental period is required to establish the business. In the initial stage of operation, since the plant is new, renewals and replacements will not be necessary for some time. To set up accruals immediately when physical conditions did not require it, or when earnings did not permit, would be unwise and, possibly, financially embarrassing, it being presumed that the project was conceived on a sound and economical basis taking into account the forecast of the growth in business and improvements in economy.

The second stage represents the time when the demand for services has rapidly increased until the territory or market has been fairly well saturated. As advance takes place in this period, the business is able to pay a fair return and accumulate a fund for retirements. Conservative management is guided by these practical conditions and gauges its reserve situation accordingly. It may speed up or slow down the rate of accumulation to the extent that it is able to foresee future changes. The percentage of reserves in relation to the investment to be protected may be maintained at a certain level for a period of time; or it may be gradually or rapidly increased as the circumstances may warrant.

When the third stage is reached and the growth of the business is definitely arrested, extra caution must be exercised. Many technological businesses follow what is termed the biological or *S*-curve as typified by the investment curve, *A*, in Fig. 35. In this day of science and invention, a business is "placed on warning" when its volume does not keep pace with the growth of the population and the markets for goods and services. If a company delays too long in accumulating reserves, in proper degree and during the productive period, for the protection of the investment, it may readily happen that it will be overtaken by disaster. When the fourth stage is reached, any attempt to overcome shortcomings of the past would be futile.

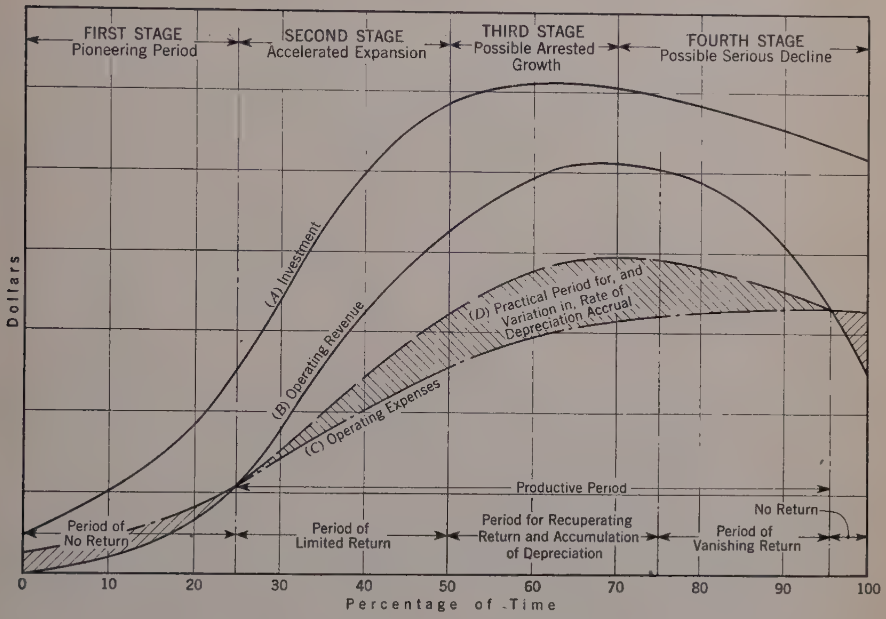

FIG. 35.—POSSIBLE DEVELOPMENTS REQUIRING CONSIDERATION IN GAGING THE ACCOUNTS FOR RESERVES FOR ULTIMATE RETIREMENT

The point that this illustration emphasizes is that as a result of various business situations the proper rate of depreciation accrual is often vastly different from that given by theoretical formulas based upon the assumed life of the items of property. The rate of accrual must vary necessarily with the productiveness of the business. It is for this reason that those responsible for management, frequently prefer to use a certain percentage of gross reserve to provide for maintenance and the annual accruals for renewals and retirements. If large sums are spent in any one year for maintenance and repairs, they have the effect of restoring the values in the property and, therefore, result in a lesser need for accruals.

Fig. 36 shows, relatively, the degree of protection provided for a given investment throughout the economic life of the enterprise under three methods of accruing for depreciation: Curve (1), Straight Line; Curve (2), Sinking Fund; and Curve (3), A Practical Method of Retirement. The first two methods do not deal with realities, and the inevitable consequences would be that they would fall far short of protecting the investment.

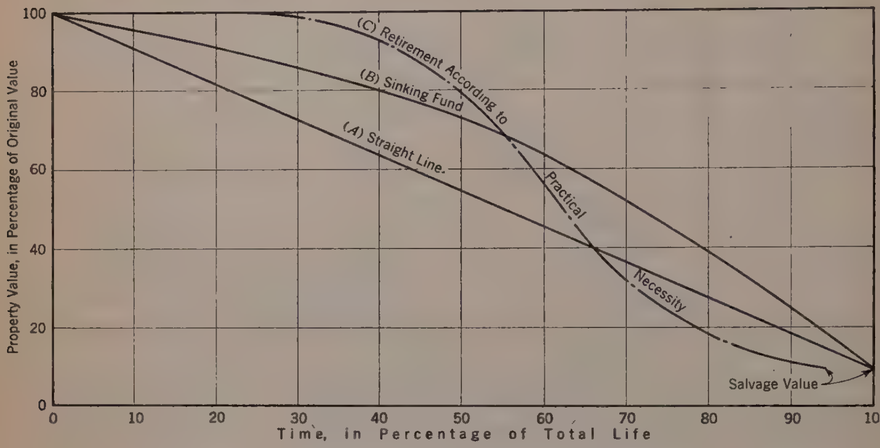

FIG. 36.—PROGRESS IN PROTECTING THE INVESTMENT IN PROPERTY BY DIFFERENT METHODS OF DEPRECIATION

The adoption of a theoretical treatment of depreciation for regulatory purposes without allowances for various practical features, may unwittingly and unlawfully deprive the owners of a substantial part of their property.

Corrections for *Transactions:* On page 652, insert "capacity" between "full" and "throughout," to read "at full capacity throughout the year"; in Figs. 10, 13, and 14, change *j* to *i* in the shaded areas; in the caption to Fig. 13, change "750°" to "825°"; in the caption to Fig. 14, change "600 lb, 825°" to "1 250 lb, 925°"; in the caption to Fig. 17, delete the word "costs." In Plant No. 4, Table 6, note the following corrected entries: Column (9), "11 years"; Column (10), "218"; Column (11), "25.0"; Column (41), "0.37"; Column (42), "0.55"; the heading for Column (10), Table 6, should read: "Average in million kilowatt-hours"; in the footnotes of Table 3 transpose the symbols, †, and ‡; in the "Relative Cost" ordinates of Fig. 8(*b*)—not Fig. 8(*d*)—change "0.40" to "0.70," "0.60" to "0.80," and "0.80" to "0.90."

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

LABORATORY INVESTIGATION OF FLUME TRACTION AND TRANSPORTATION

Discussion

By Lorenz G. Straub, Assoc. M. Am. Soc. C. E.

Lorenz G. Straub,[62] Assoc. M. Am. Soc. C. E. (by letter).[62a]—Many differences of opinion have been expressed from time to time relative to the transportation of sedimentary materials by rivers. Laboratory studies have aided in arriving at conclusions, but even in that case there is lack of agreement between various investigators. The author has presented an interesting review of several noteworthy studies which have appeared in engineering literature during the past several years; he also compares results of his own observations with results of earlier investigators. He is to be commended on his approach to the problem and his effort to give due consideration to the dimensional analysis of his theories. However, the writer cannot agree with the logic of some points in the theoretical development and the author's conclusions. He is, therefore, taking the liberty of discussing a few of these questions in some detail, without, however, giving special consideration to other features of the paper.

The data pertaining to the relation between Manning's n and size of particles are of considerable interest. They conform very closely to results obtained in connection with studies by the writer for roughness factors in laboratory experiments on sediment transportation. Of course, in case of a lack of uniformity of motion of the bed material (as, for example, when pronounced riffle formation takes place) there is considerable change in the value of the roughness coefficient. Presumably, the author's data are for relatively low velocities.

Exception is taken to the analysis of the "effect of non-uniform flow on tractive force." Equation (17) and the conclusions drawn therefrom appear to be illogical and seem to be mathematically unsound. Although the traction

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Note.—The paper by Y. L. Chang, Esq., was published in November, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1938, by Hans Kramer, M. Am. Soc. C. E.; June, 1938, by Messrs. E. W. Lane, and Joe W. Johnson; and September, 1938, by Messrs. J. E. Christiansen, and W. H. Huang.

[62] Prof. of Hydraulics, Univ. of Minnesota, Minneapolis, Minn.

[62a] Received by the Secretary August 20, 1938.

force can be written (as is usually the procedure) as a function of the water-surface slope, depth, and unit weight of the water, this conception is useful primarily as a simple means of visualizing tractive force in the case of uniform flow. Tractive force can be written in terms independent of the slope of the water surface or of the stream bottom, if due account is taken of the roughness of the bed and the velocity of the stream.[63] Subsequently, the writer will indicate one possible relation of this character, showing that for a given roughness and velocity of flow the traction force is independent of the slope of the stream bottom.

Apparently, the author assumes that Equation (15) is correct because for the special case of a sloping bed with uniform flow the usual expression for traction force is obtained. However, although the equation for the special case happens to reduce to the correct form, the general form (Equation (15)) is mathematically incorrect.

Detailed consideration will now be given to Equation (17). Assume that super-critical velocity takes place. In the case of a horizontal bed the value of S must be negative. Equation (17) would thus indicate a negative tractive force or consequent sediment transportation in a direction opposite to that of the flow of the water. Other similar illogical deductions would result from a close analysis of the physical significance of the relation. A careful analysis of the type suggested by the author for a horizontal bed will lead to the equation:

$$T = w\,S\left(y - \frac{V^2}{g}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (86)$$

instead of the expression represented by Equation (17). Here, in the case of super-critical flow (shooting velocity), S likewise becomes negative, but the expression in the parentheses becomes negative simultaneously, so that the value of T remains positive. It is impossible for flow to go through the energy critical on a horizontal bed. For this hypothetical condition in Equation (86), mathematically the value in parentheses $\left(y - \frac{V^2}{g}\right)$, would become zero and the slope, S, would become infinite; this conforms to the physical phenomenon just mentioned. In the case of non-uniform flow on a horizontal bed of infinite width the slope of the water surface in the Manning formula might be expressed as:

$$S = \frac{V^2}{C^2\,y^{\frac{1}{3}}\left(y - \frac{V^2}{g}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (87)$$

in which the value of C corresponds to the expression, $C = \frac{1.486}{n}$, in Manning's formula. Substituting this value for the slope of the water surface into Equation (87) it will be found that:

$$T = w\,\frac{V^2}{C^2}\,\frac{1}{y^{\frac{1}{3}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (88)$$

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[63] "Hydraulic and Sedimentary Characteristics of Rivers," by Lorenz G. Straub, *Transactions*, Am. Geophysical Union, Thirteenth Annual Meeting, National Research Council, 1932, pp. 375–382.

Thus, the traction force is shown to be a function of the velocity, depth, and roughness. The expression for the traction force in Equation (88) is the same as would be obtained by substituting from Manning's formula for uniform flow the value of $S = \frac{V^2}{C^2\, y^{\frac{4}{3}}}$ into the equation for tractive force for uniform flow, $T = w\, S\, y$.

The use of narrow channels in the experimental study of the effect of tractive force upon sediment transportation invariably involves certain limitations and necessitates corrections due to side-wall effects. The writer has experienced considerable difficulty in this regard even in the case of channels as wide as 12 in. Unless corrections are made for side effects the results are likely to be appreciably in error.

Reference is here made to the analytical treatment of the observations as indicated in Table 3. The side influence has been taken into account in computing the water-surface slope in Column (11). However, the computation for the tractive force (Column (13)) is incorrect, first in that Equation (17) was used and, second, in that the traction on the side walls has been neglected. Computation by Equation (17) gives tractive forces on the stream bed which are too low, while neglecting the resistance due to the sides of the channel gives an indicated tractive force which is too high. These two errors of opposite sign in this particular case have a more or less counter-balancing influence upon the result, so that the tractive force indicated in the tabulation seems to be in the proper order of magnitude. Nevertheless, if the results are scrutinized it will be observed that the computed critical tractive force

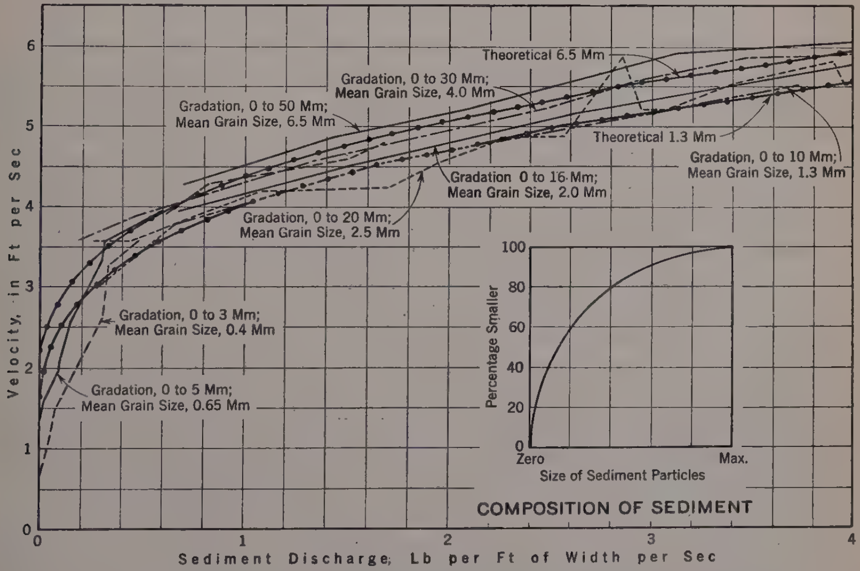

FIG. 8.—COMPARISON OF BED SEDIMENT TRANSPORTATION THEORY WITH SCHAFFERNAK EXPERIMENTS

for sediment transportation is indicated in the tabulation to be greater for the deeper streams of water. This is due to the fact that the traction resistance on the sides of the channel has greater influence as the depth of flow increases.

In connection with extensive experimental work on sediment transportation the writer has observed the phenomena noted by the author relative to an apparent change in the law of transportation for finer sediments. The change is similar to that indicated by the author; however, thus far an abrupt critical from one law to another has not been noted. A change somewhat of the type indicated in Fig. 6 seems likely although the critical point does not seem to be accurately defined by experimental results. In the writer's opinion the visual determination of traction force is likely to be unsatisfactory for use in the application of the du Boys formula for determining the quantity of material transported. It will be found that the introduction of a value for the critical tractive force or its equivalent in terms of the velocity, roughness, and hydraulic radius of the stream into some form of the du Boys formula (which is considerably different from that usually recorded from direct observation) will more nearly correspond to the actual movement of the sedimentary load. This has been previously cited by the writer in connection with studies of the sedimentary characteristics of the Missouri River.[64] It is generally found that the observed critical tractive force indicating "general motion" recorded by most observers is too low to give best results in the application of the du Boys formula. A typical example is indicated in Fig. 8, which represents the comparison of actual sediment transportation observed by Schaffernak,[65] with a theoretical analysis by the writer. The theoretical curves in Fig. 8 are plotted only for two of the series of samples tested by Schaffernak. It is interesting to note that although the experiments by Schaffernak were made with water flowing at super-critical velocities, the same theoretical analysis was applicable as for sub-critical velocities, provided due account is taken of the proper roughness factor for the particular material concerned.

The theoretical curves in Fig. 8 were computed using the following formula derived by the writer for bed-load transportation:

$$G = \psi \, C^{-4} \, d^{-\frac{2}{3}} \, V^2 \, (V^2 - V_0^2) \dots\dots\dots\dots\dots\dots (89)$$

in which C corresponds to the coefficient in Manning's formula—that is, $C = \frac{1.486}{n}$; d is the depth of flow; and V is the velocity of flow. The value of V_0 is determined from experimental data and corresponds approximately to the velocity at which transportation begins when computed by the relation,

$$T = w \, \frac{V^2}{C^2} \, \frac{1}{d^{\frac{1}{3}}} \dots\dots\dots\dots\dots\dots\dots\dots (90)$$

Thus, in Equation (89), V_0 is a function of critical tractive force as well as of the roughness of the stream bed and the depth of flow. The value of ψ is an experimental coefficient referred to as the "sediment characteristic" and is dependent upon the mechanical composition of the bed-load material; and G represents the sedimentary load. Equation (89) can also be written in other forms for use in accordance with the hydraulic factors available. The formula seems to give very good results for a wide variety of flow conditions. All the empirical constants are dependent entirely upon the mechanical composition of the sediment, provided riffle formation is not pronounced. The parameters have been determined by the writer for various types of sediment using experimental data compiled by various investigators and data collected in connection with sediment transportation studies at the University of Minnesota. In the case of the two theoretical curves superimposed upon the experimental data in Fig. 8, the characteristic values for the particular sediments were based upon the mean size of particle. Thus, for the gradation from 0 to 50 mm, the theoretical curve was computed for the diameter, 6.5 mm, and for the sedimentary material ranging in size from 0 to 10 mm, the mean size of 1.3 mm was used. For the coarser material, $\psi = 28\,000$, where G is measured in pounds per second per foot breadth of channel; $T_0 = 0.115$ lb per sq ft; and $n = 0.021$. In the case of the curve for the finer material the values are $\psi = 92\,000$, $T_0 = 0.0275$, and $n = 0.0165$. It should be noted that the critical velocity, V_0, at which sediment transportation begins is a function of the depth and roughness as well as of the critical tractive force, the latter being determined from data on rates of sediment transportation rather than simply being a visual determination of the beginning of sediment movement.

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[64] "Missouri River Report," Appendix XV, U. S. Serial No. 9829, H. R. Doc. No. 238, 73d Cong. 2d Session.

[65] "Neue Grundlagen für die Berechnung der Geschiebeführung in Flussläufen," by Dr. F. Schaffernak, 48 pp., Franz Deuticke, Leipzig and Vienna, 1922.

There are certain difficulties involved in any theoretical analysis when the riffle formation becomes a major importance. The difficulty appears to be primarily in the formulation of an accurate value for the roughness coefficient. This is particularly true in the case of shallow depths of flow. There are different modes of transportation when riffles form. Three types of bed-load movement can be clearly distinguished. In one instance, riffles progress forward after the fashion of sand dunes, practically all particles coming to resent periodically when reaching the leeward side of the dune. For this case the writer has found that the bed-load can be computed readily by simply taking into account the shape, height, and velocity of the riffles. Another extreme is the condition under which no riffles form—that is, when the bed is quite smooth. For this case the roughness coefficient has been reasonably well fixed and the bed-load can be computed with a reasonable degree of accuracy. There is an intermediate condition between these two extremes in which undulations or riffles of a rounded form appear on the surface of the bed. Here there is a progressive movement of the riffles, but there is also general movement of the stream bed. Computation based solely on the height, shape, and speed of the riffle in such instances is found to give a sedimentary load less than the actual load. These different formations are related at least in part to the Froude number for the condition of flow. Thus, for a Froude number of unity or flow near the energy (velocity) critical, very pronounced dune-type riffles will frequently be observed with certain types of sand. The mechanical composition of sand also influences the character of the sediment transportation.

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DISCUSSIONS

FLOOD ROUTING

Discussion

By Messrs. William T. Collins, George E. Townsend, and Waldo E. Smith

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William T. Collins,[41] Jun. Am. Soc. C. E. (by letter).[41a]—The influence of volume occupied by vegetation and of ground storage on the amount of storage for flood routing does not nullify the usefulness of volumes found from contours. This fact has been demonstrated by the authors. Heretofore, no study using that method to determine storage has been published as far as the writer knows, either because of lack of sufficiently accurate topography and data on profiles, or for lack of the perspicuity to realize that that approach could be used. To obtain the amount of storage from contours probably involves more labor than to build it up from differences of inflow and outflow hydrographs. Establishing the profiles necessary in each reach is a major hydrologic problem in itself. To those engineers who do not have the time or the data necessary to determine storage in this manner, a comparison of these storage volumes with those that might have been determined from differences of hydrographs, would be extremely interesting. Probably storage determined from hydrographs is absolutely accurate if the hydrographs are correct. Because of conditions peculiar to a flood, hydrographs are often not correct unless they are determined from rating curves based on discharge measurements made during that flood. Rating curves are usually based on measurements taken at various times; therefore, hydrographs and storage determined from them are not absolutely correct, and it would be desirable to compare storage determined in this manner with that determined from contours.

The profiles of non-steady flow are an original and logical approach to determining volumes under conditions which prevail most of the time during floods. Until now the best method of computing storage under such conditions

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Note.—The paper by Edward J. Rutter, Assoc. M. Am. Soc. C. E., and Quintin B. Graves, and Franklin F. Snyder, Juniors, Am. Soc. C. E., was published in February, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1938, by Messrs. Cecil S. Camp, and R. D. Goodrich; and September, 1938, by Messrs. E. L. Myers, and Ralph W. Powell.

[41] Junior Asst. Engr., Muskingum Watershed Conservancy Dist., New Philadelphia, Ohio.

[41a] Received by the Secretary August 17, 1938.

was using storage volume as "equal to the length of the reach times the average end area."[42] This is probably all right as long as the stream remains entirely in its channel; but as soon as the flood goes over-bank, it is difficult to imagine that the end, or any sections, is typical. Gaging stations are usually located where the over-bank flow is relatively restricted, which tends to make this relation even less valid. Possibly a solution would be to find the storage from hydrographs and plot it as a function of end area.

In any reach where there is a major tributary, the authors assume that distribution of discharge from the head to the foot should vary with drainage area. Consider such a reach where the stages are rising—in it inflow will exceed outflow, and discharge at the head of the reach may also (see, for example, December 22 in Table 3(*a*)). Under this circumstance the authors step discharge down at the mouth of the tributary when actually it would step up, for the tributary will always be adding some discharge. During the falling stages their assumptions closely approximate the actual discharge profile. A different treatment of local and tributary discharges might improve the method.

Tributary and local flows can be routed to the foot of the reach by adjusting time lag and distribution coefficients, or it would be possible to find the total storage and separate it according to whether it affects flow that enters at the head of the reach or that which enters along the reach. Straight-line distribution of discharge in a reach will suit conditions better, provided the stations are close enough together for the sharpness of the flood wave, if tributary and local flows are treated separately. With this plan, the discharge at the end of a reach would be the outflow as routed from the head plus local and tributary discharges. The steps in the discharge profile would come at routing stations where they are easily handled. Total storage would not be obtained under this treatment of tributary flows, but it would be approximately the storage that affects the water entering at the head. Since tributary water enters at some point other than the head, it should be treated differently from that which does enter at the head. Consider the discharge profile of a reach under two conditions: First, when head inflow is greater than outflow during a rising stage and tributaries are not adding materially to the inflow, the profile may be considered a straight line; and second, during a faster rise, when tributaries are adding, the profile will no longer be considered a straight line, but will have an increment, and, in most cases, a change of slope wherever a tributary enters. If the head inflow in the second condition is the same as before, if the outflow of water that entered at the head is less than in the first case, and if the tributaries have added enough so that the total outflow is equal to that in the first case, the authors' method would give the same storage in the two cases. The storage in the second case may be less, equal to, or more than, in the first case, depending on the position of the tributaries and whether or not they have risen or fallen since the time that their discharge influenced the present outflow. These conditions, which are redundant with the present treatment, would not be so if tributary and local flows were routed separately.

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[42] "The Hydraulics of Flood Movements in Rivers," by Harold A. Thomas, M. Am. Soc. C. E., *Engineering Bulletin,* Carnegie Inst. of Technology, 1934, p. 54.

It should not be taken as general, that "the distribution graph could be somewhat in error without materially affecting the discharge computed for the main stream." On the Muskingum River, the drainage area at Zanesville, Ohio, is 6 839 sq miles, and the local and tributary area from Dresden, the station next above, is 857 sq miles, 12% of the whole. The crest at Zanesville is in some cases caused by the tributary which is flashy, and at those times occurs when the tributary adds its sharp crest to the relatively flat hydrograph of the main stream. The tributary may add as much as 35% to this crest, so that in this particular case it is necessary to know the tributary accurately.

The Muskingum Watershed Conservancy District uses curves similar in purpose to those of Fig. 9. It is realized that the outflow influences the regulated profiles of Fig. 4. However, it is easily seen that the flow at the head of the reach has more weight in determining the amount of storage. The curves made up by the District are for various uniform flows throughout the reach, and since the effect of variation of outflow from inflow is ignored they are drawn in permanent form. There is one other difference, the District being primarily a flood-control agency its interest is in the storage available for flood reduction, and its curves show what is called net reservoir storage, which is the difference between actual storage for the range of reservoir water-surface elevation at each of the chosen flows, and the storage for the same flows in the natural state without the dam.

The accuracy of the authors' method is stated to be satisfactory, but no comparison with other methods or percentage of this accuracy is given. It might have been advisable to route to the stations at which hydrographs of the floods studied were available, as well as to the stations at the dams where the main interest is. This would allow comparison of computed with actual hydrographs for the same stations.

George E. Townsend,[43] Jun. Am. Soc. C. E. (by letter).[43a]—In this flood routing paper, the authors state that the time period is 24 hr, but that in routing on flashy streams a shorter time period is required. When any inflow hydrograph shows a sharp change from rising to falling stage, or when it shows marked differences from day to day, a shorter period should be used in order to give more accurate results. The necessity of a routing having shorter time periods is definitely shown when a greater peak outflow than peak inflow is found when using a longer time period. The peak outflow cannot be greater than the peak inflow because, on the rising side of the hydrographs, the stages are rising and a part of the inflow goes into storage. As long as some of the inflow goes into storage, the outflow will always be less than the inflow, until the peak of the outflow is reached. The condition is unusual and can be eliminated by the use of the shorter time period.

Fig. 12 shows a comparison between routings using the 24-hr and the 6-hr time period. The reservoir projects were operated at normal depressed winter eleva-

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[43] Junior Hydr. Engr., Tennessee Valley Authority, Knoxville, Tenn.

[43a] Received by the Secretary August 25, 1938.

tions for each of the main Tennessee River dams, and all the flow was stored at the tributary reservoir projects above Knoxville, Tenn. The hydrographs apply to the Chickamauga Reach. The use of the 24-hr period gives an outflow about 11 000 cu ft per sec greater than the inflow. The application of the 6-hr time period results in an outflow peak discharge slightly less than the inflow and occurring 12 to 18 hr before the peak, as determined in the 24-hr routing. In addition, the outflow hydrograph is crossed by the inflow hydrograph nearly at the highest point of the outflow hydrograph, a criterion which usually may be applied as one check upon the results. The differences in the computed outflow hydrographs using the two different time periods may be accounted for by the impossibility of determining any flow profile to fit all flow conditions, and to the difficulty of determining an accurate local inflow. Upon the flow profiles, of course, depend the storage curves which are used in the routing. The method used in obtaining the local inflow gives good average results, but its determination depends on taking into account the water losses, the season of the year, and the condition of the ground during the particular flood. The results, therefore, are not accurate.

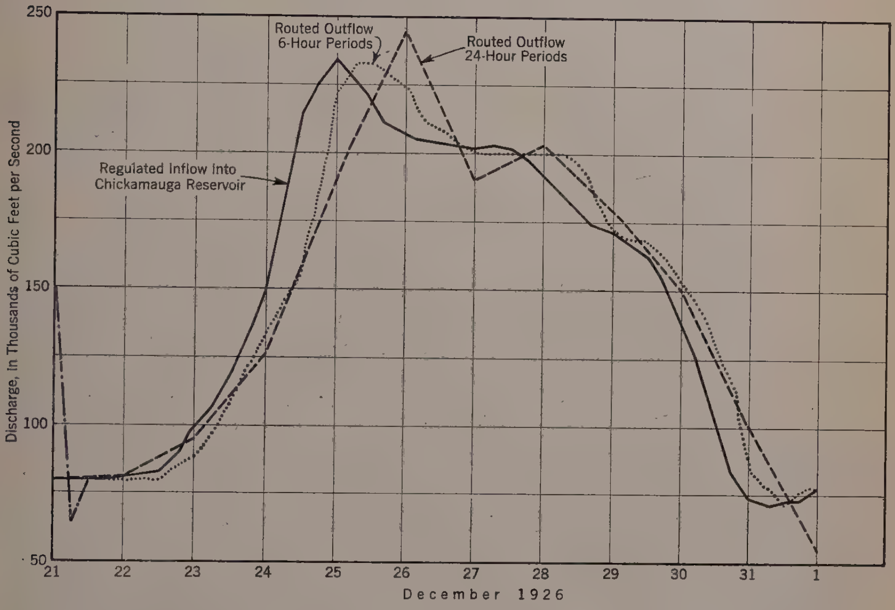

FIG. 12.—COMPARISON OF 1926 FLOOD HYDROGRAPHS AT CHICKAMAUGA DAM SITE USING 24-HOUR AND 6-HOUR PERIODS IN ROUTING

The method of using the shorter time period when it is required does not entail much extra work. It can be accomplished with the same sets of storage curves used in the 24-hr routing. Since available discharge or stage records often indicate the average day's flow or the rate of flow at a certain hour, it is advisable to convert these records into discharge curves or hydrographs.

Then, from the curves, or hydrographs, the probable rate at any hour or the total for any period may be taken. The flow into the upper end of the reservoir and the local and tributary flow into the reservoir are adjusted in that manner. The sum of these two flows for each period is the flow at the beginning of each period, termed I_1, in the paper. Then all values of I_1 are divided by 4 if the time period desired is 6 hr. Next, the outflow at the beginning of the period (O_1) is found in the usual manner, but it is converted to $\frac{O_1}{4}$ so as to find the storage at the beginning of the next period (S_1) and to find $O_2 + S_2$. Several other methods may be developed, but this one may be applied to both natural conditions and to those with regulated conditions. If the storage curves for the natural conditions, however, are in the form of "outflow-plus-storage," these curves must be put in the form of "storage" alone before this method may be applied.

Also, in Fig. 12, is shown the use of two different starting flows and their result on the computed outflow hydrograph. The lower starting flow is one in which the outflow is assumed equal to the inflow to start the routing which is usually just about the actual condition at the start. The other starting point, the higher flow, can be seen to dip sharply immediately in the first 6-hr period used; but the satisfactory part about it is the rapidity with which it gets back to the original outflow hydrograph with its 6-hr period. This demonstrates that practically any outflow may be assumed to start a routing and that, regardless of which one is used, the critical peak point is not affected. Furthermore, it shows that small errors in curve reading are compensated for within several periods in the routing procedure.

A simple check on the arithmetic of a routing computation is convenient. First, all the inflows at the beginning of each period are added. Then a "correction" is determined which, when applied to the summation of the inflows, should equal the summation of all the outflows. To find this "correction" the inflow, outflow, and storage are considered for the first and the last periods of the routing computation. The first and last periods are used because the actual routing computation involves the period between the middle of the first and the middle of the last periods. Actually, the first half of the first period and the second half of the last period do not enter into the total volume of the routed hydrograph. The difference between inflow and outflow, in which the unit is in day-second-feet, therefore, is divided by 2. This is done for both the first and the last period. The difference between storages, in which the unit is acre-feet on the first and the last period, is divided by 2 to convert it to day-second-feet. The sign applied to the flows is negative when the inflow exceeds the outflow. If the storage in the last period is larger than in the first period, its sign is negative. Finally, the algebraic sum of these three corrections is applied to the summation of all inflows at the beginning of each period, and this should equal the summation of all the outflows.

Using Table 3(*a*) the correction for December 21 is $37 - 45 = -8$ day-sec-ft; for December 31, it is $211 - 170 = +41$ day-sec-ft; and for the storage correction the difference between December 21 and 31 is 48 − 291 = − 243 acre-ft. Then, $\frac{-8 + 41 - 243}{2} = -105$ day-sec-ft; and,

$$\begin{aligned} \Sigma I_1 &= 2\,045 \text{ day-sec-ft} \\ &\ \underline{-\ 105} \text{ day-sec-ft} \\ \Sigma O_1 &= 1\,940 \text{ day-sec-ft} \end{aligned}$$

which checks.

Possibly, some improvement may be made in the manner of determining a set of storage curves for the Tennessee River. Another method might take care of bank storage and water losses, where the authors' method does not, because storages were determined through the use of topographic maps. The manner in which the flow is varied from the upper end of the reach to the lower end might be changed so that better routing computations would result. Good results are obtained in reaches where a relatively large tributary enters near the middle of the reach, and the flow is varied on the basis of the drainage area. In reaches where no large tributary enters and the flow is varied directly with the length of the reach, results are only fair. Floods in which there is a sharp peak do not work out as well in reaches where the storage curves were made up by varying the flow with the length of the reach. Floods with a peak of long duration seem to yield better results; and, better results might be obtained if the flow were varied by a curve instead of a straight line between the upper and lower end of a reach in which no large tributary contributes its flow.

WALDO E. SMITH,[44] ASSOC. M. AM. SOC. C. E. (by letter).[44a]—The authors are to be commended for their splendid contribution to hydraulic literature pertaining to the routing of floods. The writer is pleased to note that their method abandons completely the old conceptions of prism and wedge storage and likewise circumvents the seemingly insurmountable step from theoretical and approximate formulas for uniform slopes and channels to their application to actual channels of varying section and slope. Furthermore, after the preliminary work is complete and the basic curves are computed and plotted, the method is expeditious and is satisfactory for use for analytical purposes especially, and for limited use in forecasting and in operation for the flood control of a reservoir system applied to the natural channel and channel reservoir reaches.

Flood-routing problems, quite naturally, resolve themselves into two quite distinct although overlapping parts, the first of which, in turn, is further divided into two aspects, as follows:

(1) The routing of floods in natural channel and channel reservoir reaches:
    (a) For analytical purposes, such as the critical survey of the operation and routing undertaken during the progress of a flood; and,
    (b) For forecasting and flood-control operation of reservoirs.

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[44] Hydr. Engr., Muskingum Watershed Conservancy Dist., New Philadelphia, Ohio.

[44a] Received by the Secretary August 31, 1938.

(2) The routing of floods through comparatively deep, broad, valley reservoirs in which the back-water transition curve is a negligible item in the volume of flood storage available. (The latter may likewise be subdivided but that subdivision is not pertinent to the subject as developed by the authors.)

The work of the authors is especially applicable to Problem 1(*a*). It can be used in forecasting Problem 1(*b*) also, but only to the extent that the upper end of the uppermost reach may be forecast.

The authors mention that the time of travel through the reach should usually be about the interval used in the analysis and that, in most cases, this appeared to be about 24 hr. The Muskingum Watershed Conservancy District has a method under study and in partial use that facilitates forecasting to successively later times as progress is made down stream. Assuming the period of forecasting to be for 24 hr, the technique developed by the District bases the forecasting for the lower station on the inflow into the reach at the present time, the anticipated storage in the reach 12 hr hence, and the local inflow 24 hr hence by the distribution graph method. Upon the completion of a storm, knowing the run-off factor, the forecaster can complete his work for the remainder of the flood period. By the District method it has been found, if the time of travel is 24 hr, that 12, 6, or 4-hr intervals can be used in routing as satisfactorily as 24 hr. The time interval use should never be greater than the so-called time of travel through the reach but it may be less. The development of outflow-plus-storage curves appears to have the effect of limiting the use of these curves to 24-hr periods. In the District work on analytical studies, the stage at the lower station has been based on estimating the outflow at the same instant of time that the inflow is known, similar to the work outlined by the authors.

The writer acknowledges that there is a substantial difference between the problems on the Tennessee Valley and those of the Conservancy District. In the former the upper end of the uppermost forecast reach drains about 9 000 sq miles and the routing is carried through a system of reservoirs that are essentially channel reservoirs. The total drainage area involved in the Conservancy District, on the other hand, is only 8 000 sq miles and the reservoirs are largely head-water reservoirs with storage capacities relatively large in comparison to the water-shed above; and routing and forecasting on the main streams are almost entirely in natural channels. The forecast method used by the District is found not to be uniformly applicable to all reaches, and, in some reaches, in which a large tributary enters, it has not yielded a satisfactory solution. It has been necessary to use a number of variations in the general procedure.

The authors were fortunate indeed to secure contour maps of the channel and valley floor which gave them the opportunity to determine storage in 2-mile reaches by means of contours and the profile of the stream. They were fortunate even to have cross-sections at sufficiently representative and frequent intervals. Such maps and cross-sections are not usually available and one must rely on the hydrographs to find the valley storage at any particular time for a past flood, for the development of the storage curves. This has the disadvantage of involving the local inflow and its distribution with regard to time as well as valley storage and the difference between the two hydrographs at any particular time is thus a function of two independent variables which can only be determined by trial and error. The volume of local inflow is determined by means of the hydrographs as outlined by the authors; but one must realize that this throws the entire cumulative error in rating curves into the local inflow which is usually the smallest part. The procedure on the work of the Muskingum Watershed Conservancy District has been to assume an apparently reasonable distribution graph and determine the valley storage in a reach for a given flood on this basis. A storage curve, usually with a hysteresis loop, is found and smoothed up, and the distribution coefficients are modified accordingly. Analyzing a second flood of similar magnitude, using the same distribution graph, the new storage relationship is determined, which usually differs from the first. The distribution graph is then adjusted to bring the two curves as closely together as possible and the resulting channel storage relationship is used. Local inflow in all cases has been applied to the lower end of the reach, however, instead of at the upper end as has been done by the authors. This has the tendency to change the characteristics of the distribution graph considerably, giving a relatively small percentage of run-off for the first day (or other time interval) and a much larger percentage on the second day. The effect is substantially the same, but the technique used by the District makes it convenient to find the discharge at the lower end of the reach and then add the value for the local inflow to establish the forecast gage height.

One of the difficulties of the old prism and wedge storage idea was that it failed to take into account the fact that the flood crest, or the trough between crests, is sometimes between the limits of the reach. Either condition alters prism and wedge valley storage substantially. The authors seem to offer no means of overcoming this difficulty inasmuch as the method assumes a constant rate of change of inflow, outflow, and, thus, the storage rate for each 24-hr routing period. The same problem has been a stumbling block in some cases with the forecast method used by the District outlined briefly; but by using a shorter interval than the time of travel in the reach it has been possible in some instances to compensate for it satisfactorily.

The back-water transition curves in Figs. 3 and 4 scarcely appear to be typical even of channel reservoirs. If the straight lines indicated for the profile are chords of the actual curve and were used as shown, it appears that a large thin volume of storage is included between the Chickamauga Dam and the Hiwassee River that is not actually available because of the tendency for the profile in the reservoir to be concave upward and to be nearly horizontal at the dam.

There is one question that the writer raises in connection with the speeding up of the flow through the channel reservoir. The authors have indicated that they believe that the fact that it takes less storage to increase the flow a given amount down stream compensates satisfactorily for this. The writer is not sure that this is complete compensation if the time of travel is decreased substantially. The writer would also like to know why, in the first instance, O_1 must be estimated or assumed if there is a gaging station at the lower end of the reach?

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

MOTOR TRANSPORTATION—A FORWARD VIEW

A SYMPOSIUM

Discussion

BY BRUCE D. GREENSHIELDS, ASSOC. M. AM. SOC. C. E.

BRUCE D. GREENSHIELDS,[26] ASSOC. M. AM. SOC. C. E. (by letter).[26a]—The least understood but perhaps the most important problem of motor transportation is that of harmonizing highway construction and motor-vehicle design with human behavior.

The engineer has been building highways for hundreds of years. He has learned how to deal with inanimate materials and forces that behave in a definite manner under given conditions. Human nature seems perverse and is so uncertain that the engineer, having correctly designed a highway from a mechanical standpoint, is inclined to let the "fools" kill themselves if they insist upon it.

Whatever his inclination, however, the engineer must accept the fact that the people who use the highways pay for them and, therefore, have a right to demand that they be safe. Each driver should be held to account for his own mistakes, but he must be protected from the mistakes of his neighbors on the road.

Mr. Kettering states, in the "Synopsis," that "the vehicle and the road must form a complete transportation system" and that "each one is dependent upon the other for the success of the whole." The writer would add that the whole must include the driver. Not only is highway safety dependent upon the driver's ability, but its utility is limited by his whims and desires. One loitering motorist may hold up a long line of vehicles.

There is ample evidence that drivers are foolhardy. Recently the daily press,[27] quoting from a pamphlet issued by the Board of Railway Commissioners

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NOTE.—This Symposium was presented at the meeting of the Highway Division, Detroit, Mich., July 21, 1937, and published in June, 1938, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: September, 1938, by Messrs. F. Lavis, Edgar Dow Gilman, George Hartley, Robert Kingery, R. L. Morrison, and Roy F. Bessey.

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[26a] Received by the Secretary August 17, 1938.

[27] *The Kansas City Times*, August 8, 1938.

of Canada, asks why so many people risk their lives by trying to beat the train to the crossing:

> "The latest safety devices and cautionary signals may be in operation, yet to a strange element of humanity these appear to present only a challenge.
>
> "A summary of the dangerous practices at railway crossings only adds to the wonderment as to why many drivers behave as they do: of all the accidents and near accidents reported (350 drivers and pedestrians in 6 months) 22.3 per cent of offenders ran into lowered gates, 18 per cent crossed in front of approaching trains, 11.7 per cent drove under gates while being lowered, while the remainder resorted to utterly foolish ways of risking their lives."

The people who deliberately commit suicide on the highway are few indeed. The explanation lies in another direction. One reason is that the driver or the pedestrian does not know the risk he is taking. Statistics show that a large percentage of injured pedestrians do not drive cars. They put undue trust in the ability of the driver to see a pedestrian and to bring a car under control. If one's attention is momentarily distracted he may not see something that is otherwise perfectly obvious. Every one has had such an experience. The driver has had little more than a generation within which to learn to avoid the unnatural hazards of high-speed cars and to adjust himself to the unnatural experience of driving.

Professor Yandell Henderson speaking before the National Academy of Science, at Charlottesville, Va., in November, 1935, stated that:

> " 'The car went out of control' is a usual explanation of automobile fatalities, but in reality, it is the driver who goes out of control. He acts in the only manner that his nature permits, the manner in which every human being always instinctively acts and always will act under the circumstances."

Any motorist, no matter how experienced, according to Professor Henderson, when suddenly and severely jolted, instinctively re-acts to steady himself in his seat, and in doing so, presses his foot down on the accelerator pedal. The more he bumps along the harder his foot presses and the more rigidly he grasps the wheel. People cannot be changed but motor cars can. Professor Henderson would have a pedal under the left foot which would counteract the effect of the right by shutting off the power.

There are many phases of behavior—physical, mental, and emotional—that need to be considered, but obviously only a few can be considered in a short discussion. One of the natural attributes most discussed in connection with automobile driving is that of "reaction-time." This is the brief interval between the happening of something in one's environment and the time when a certain response is made to it. This interval may be, for example, that between the time a pedestrian steps into the street and the time the brake is applied to avoid hitting him.

In general, the speed of all forms of reaction varies from one person to another and from time to time in the same person. Extreme fatigue decreases the ability to respond. Emotions such as fear or anger interfere with the driver's reactions. The effect of alcohol varies with the individual—it does not necessarily lengthen reaction-time, but an excess amount makes the reaction-time more uncertain. The reaction to hearing is quicker than to sight.[28]

Certain responses are made more quickly than others. The hand is quicker than the foot. Perhaps a hand-operated brake would be better in an emergency.

Although reaction-time has been considered mainly in connection with highway safety, a means for reducing it is probably more important in highway design than in safety work because it has not been demonstrated that reducing reaction-time would lessen accidents. On the other hand, it would speed up traffic and increase highway capacity.

Persons who react more quickly drive at higher speeds. They thus void whatever advantage their quickness gives them and have just as many accidents as slower drivers.[29] The average minimum spacing between vehicles depends upon the reaction-time of the individual drivers. The spacing between vehicles fixes the number that can occupy any length of roadway and, hence, is one of the factors determining highway capacity.[30]

Earl Allgaier, Safety and Traffic Engineer of the American Automobile Association, after subjecting approximately 22 000 drivers to a battery of 15 tests, states that driving ability reaches a maximum at about 25 yr of age.[31] The decline begins between 40 and 50 yr and grows rapidly from that point on. The tests included field of vision, eye dominance, excitability, blood pressure, hearing, and steering ability. Mr. Allgaier states that the falling off in the test scores made by drivers more than 40 to 50 yr of age need not necessarily indicate that these drivers are actually poorer or less safe than younger drivers. They are more experienced and likely to be more cautious.

Dr. H. M. Johnson, referring to such tests, states that he has found no published report of a comparison which warrants the conclusion that test performance is closely associated with accident rate.[32]

Dr. A. R. Lauer, of Iowa State College, who has developed many of the tests, states[33] that after having analyzed several sets of data it is his guess that "when our final analysis is made we will find no high validity coefficient." He adds as a reason the fact that "human nature is unstable and highway conditions are highly variable." There is significance for the highway designer in the last statement. Human nature cannot be changed but highway conditions can be made less variable.

To claim that human nature cannot be changed does not mean that one cannot learn to do things better or that natural handicaps cannot be overcome. The quickness with which one can move his foot in a certain way in response to a given stimulus cannot be reduced by practice beyond a certain point; but by simplifying the movement required to control the vehicle the car manufacturer can, in effect, shorten the reaction time. Although there is a natural limitation to one's ability to see, cars can be made easier to see out of and highway signs can be improved by an increase in size or by illumination.

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[28] "Reaction-Time and Traffic Behavior," by Bruce D. Greenshields, *Civil Engineering*, June, 1937, p. 384.

[29] "A Study of Factors Influenced by Automobile Brake-Reaction Time," by Frank R. Olmstead, *Proceedings*, 22d Annual Highway Conference, Ann Arbor, Mich.

[30] "Studying Traffic Capacity by New Methods," by Bruce D. Greenshields, *Civil Engineering*, May, 1935, p. 301.

[31] *Public Safety*, May, 1938.

[32] "The Detection of Accident-Prone Drivers," *Proceedings*, Highway Research Board, 1937, p. 445.

[33] *Loc. cit.*, p. 452.

The experienced driver has learned to make the proper responses to a large number of situations. He is less likely to be surprised. Experience enables one to make quicker judgments. Judgment is more complex than perception. For example, in taking a laboratory test of brake reaction-time, a light is flashed and the subject presses the brake pedal. He perceives the signal and acts. No judgment is involved. He has decided what to do beforehand. In the case of passing a vehicle on the road, he first sees the vehicle approaching on the opposite lane. He then decides whether he can pass the car ahead. This deciding is a matter of judgment and is a much more complex process than "sensation." It may take minutes instead of seconds.

The young driver cannot be given experience but experience can be made less necessary by simplifying the highways and by informing the motorist of approaching conditions so that he has time to decide what to do.

Consider highway curves, for example; in most States all curves are heralded by the same type of sign. The driver does not know whether the curve is sharp or flat or whether he can drive it at 50 miles per hr or whether he should slow to 20 miles per hr. Why not give the motorist the information he needs?

Common arguments against such a practice are that the safe speed for one car is not that for another car and that the State would be to blame if a defective or old-model car had an accident while traveling at the announced speed. The curves should be designated by numbers as 1, 2, 3, etc., to indicate riding qualities. The motorist would soon learn at what speed his car would take a No. 1 or a No. 2 curve.

Adequate highway signs might have prevented two fatal accidents that occurred at Fidelity Corner, six miles south of Carthage, Mo., in August, 1938. In the writer's opinion the rather inconspicuous "Slow" sign, followed by another indicating the intersection, should be replaced by (1) a prominent intersection sign, and (2) a danger sign; and, to make sure, a "blinker" should be added. Note that, first, the driver would be informed of what to expect; and, second, would be warned of danger.

Highways must not be deceptive. Many a new highway has failed to reduce accidents. Its deceptive appearance of safety has induced motorists to step up their speed to a point at which they did not have time to react to the road's surprises.

Speaking on the subject of "Highway Design and Safety" at the Highway Safety Conference, Yale University, on April 4, 1938, Guy Kelsey, of the Signal Service Corporation, Elizabeth, N. J., described the surprise hazards confronting drivers operating on 5.4 miles of a "very modern highway" selected at random. Low visibility, head-on-intersection-of-vehicle hazards, as well as several traffic flow inefficiencies were among the deficiencies found in the short stretch.

The statement is made by Mr. A. R. Nichols[34] that even if the safety and utilitarian requirements have been met, the road that does not closely follow the lay of the land results often in glaring artificiality of ugly cuts, high fills, and an increased area of destruction of natural vegetation. Such unnecessary artificiality makes a road more dangerous. Its unnaturalness leads to surprises.

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[34] "Highway Design: Its Relation to Landscape Objectives," by A. R. Nichols, *Proceedings*, Highway Research Board, 1937, pp. 269–278.

Although the unexpectedness in human nature cannot be removed like the perplexities of the highway there is the gratifying fact that human nature *en masse* is not so variable after all. Insurance companies cannot tell how long any individual will live, but they have built up a thriving business on predicting how long the individuals in a large group will live.

From his studies of behavior on the highway (such as the spacings maintained between vehicles, the distance required for passing, and the point at which drivers slow down because there is a vehicle ahead) the writer has arrived at the definite conclusion that drivers do behave in a definite manner under certain conditions. The curves resulting from the plotting of such data are surprisingly smooth.

The traffic engineer must study human nature as the design engineer has studied soils and concrete and steel. Road intersections that are unsafe must be compared with those that are safe to find out how they differ. The "specifics" of driving behavior must be determined by careful analysis. How prominent must a sign be for the driver with poor eyesight to see it? Advertising signs are much more prominent than highway signs. At what distance can one determine whether or not an object is moving? These are the facts the motorist needs to know, and does not.

The highway of the future must expedite traffic and it must do it safely. It must be honest. If it invites speed it must have no dangerous surprises. As sight distances are increased and vehicles are made even more responsive to control, speeds will go up in direct proportion. If 50 miles is a high average speed now it is because drivers feel comfortable and safe at that speed. If average speeds of 100 miles per hour are adopted in the future it will be because the driver feels safe at that speed. When speeds on limited ways go high enough they will be restricted to the best drivers.

It is hoped that these remarks have added something to the excellent papers of the Symposium by emphasizing the importance of studying the highway user.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

PRELIMINARY DESIGN OF SUSPENSION BRIDGES

Discussion

By A. Fraser Rose, M. Am. Soc. C. E., and
William A. Rose, Esq.

A. Fraser Rose,[31] M. Am. Soc. C. E., and William A. Rose,[32] Esq. (by letter).[32a]—This timely paper is quite in line with the modern trend in structural analysis and design—simplification through successive approximations—and is a worthy addition in this field to the method of moment distribution.[33] The proposed method is developed in such a manner that it possesses the added advantage of affording the designer a clear mental picture of the contributions to the final results made by the various factors.

The great difficulty in analyzing a structure of the nature of the stiffened suspension bridge is that superposition of results is not permissible, inasmuch as they are inter-related. The authors have overcome this obstacle by the clever device of considering the contributing factors in their relative order of importance, and subsequently correcting for the errors introduced by trial superpositions.

As cited in the paper, Hardy Cross, M. Am. Soc. C. E., has proposed a logical classification of structures[4] which tends to clarify the relation of analysis to design. Four types of structural actions are emphasized: (1) The action which produces deformation stresses; (2) that which produces participation stresses; (3) normal structural action; and (4) hybrid structural action. The proposed method clearly indicates that the stiffened suspension bridge exhibits characteristics of all four of these types of action, but that it is primarily a

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NOTE.—The paper by Shortridge Hardesty and Harold E. Wessman, Members, Am. Soc. C. E., was published in January, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1938, by Messrs. A. W. Fischer, and Jacob Karol; June, 1938, by Messrs. Glenn B. Woodruff and Norman C. Raab; and September, 1938, by Messrs. Hardy Cross, and A. A. Eremin.

[31] Cons. Engr., New York, N. Y.

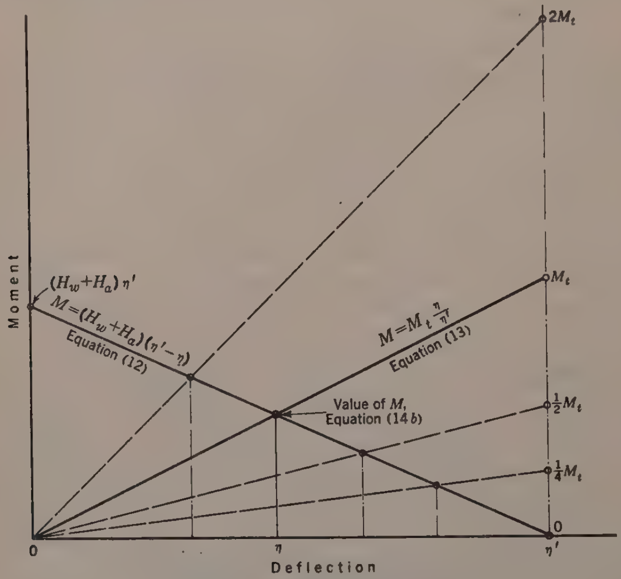
[32] Asst. Prof. of Structural Eng., New York Univ., New York, N. Y.

[32a] Received by the Secretary August 22, 1938.

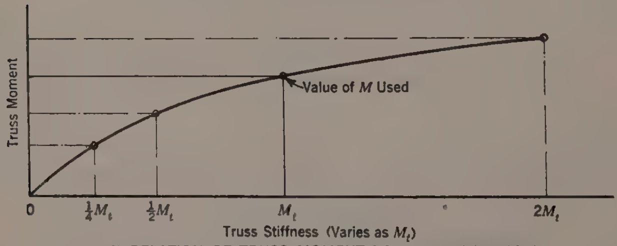
[33] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

[4] "The Relation of Analysis to Structural Design," by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1363.

hybrid structure. As the authors point out, there are any number of possible designs for the stiffening truss in a given instance. Any change in truss stiffness is reflected in a change in the moment resisted by the truss, a typical hybrid action. Where hybrid action exists, preliminary designs for comparative study are profitable and enlightening.



(a) RELATION OF MOMENT AND DEFLECTION FOR VARIOUS TRUSS STIFFNESSES



(b) RELATION OF TRUSS MOMENT TO TRUSS STIFFNESS

FIG. 14.—EFFECT OF TRUSS STIFFNESS ON MOMENT, M, AND DEFLECTION, η (NEGLECTING CABLE STRETCH, ETC.)

The use of exact, or relatively exact, methods of analysis for preliminary studies is not justified. The small added degree of accuracy attained consumes an unwarranted length of time and serves no useful purpose. The importance of developing, on rational bases, approximate methods for rapid analysis in such cases cannot be over-estimated. Their availability is conducive to a proper study of the effects on the analysis of the variation of the several factors which enter into the design. In this way, the engineer is enabled to secure the maximum economy consistent with stability.

The writers agree with the obvious conclusion of the authors that the determination of the maximum shears is not essential for preliminary design purposes. One finds in designing the web members of the stiffening trusses that minimum section rather than stress governs the areas in many cases. Even in the final design, maximum shears are needed at only a few points; for the preliminary work, sufficient information may be obtained from the maximum moments at critical sections.

A number of interesting conclusions may be reached after careful inspection of the equations that enter into the calculation of the moments at a critical section. For example, consider Equations (14*b*), (24*a*), (37*a*), (40*b*), and (42) in their relation to the determination of maximum moments at the quarter-point of the main span.

The final total moment at the quarter-point is the sum of the values obtained from Equations (14*b*) and (37*a*). In the first of these values, it is evident that an increase in M_t, which from Equation (24*a*) is proportional to I, results in a less than proportional increase in M, due to the fact that M_t appears in the denominator as well as in the numerator of the expression on the right-hand side. This is clearly shown in Fig. 14, in which the final moment is the ordinate of the point of intersection of the lines, $M = (H_w + H_a)(n' - n)$, and $M = M_t \frac{n}{n'}$ (Equations (12) and (13)).

Fig. 14 and Equation (14*b*) emphasize the existence of hybrid action. Hybrid action is further shown by inspection of the factors that contribute to the change in sag, df. From Equation (37*a*), it might hastily be concluded that df should vary inversely with I. This conclusion tacitly assumes that M would remain fairly constant. Careful study reveals that M, due to this cause, increases almost as rapidly as I, so that the decrease in df is considerably less than proportional.

Three factors contribute to df. Temperature change contributes a constant deformation in the cable and a constant change, df, in sag, regardless of truss stiffness, except for a small correction due to the additional stress in the cable caused by the temperature change. This action by itself then comes under the first of Professor Cross' classification—action which produces deformation stresses.

The second factor is the change in cable stress from that under dead load at mean temperature, caused by the addition of live load; this contributes almost a constant amount to df except for small changes due to its own action.

The third factor is the element termed side-span interaction. The contribution to df from this cause is by no means constant, but is affected by any change in side-span stiffness. This is brought out in Equations (40*b*) and (42), and in Table 2. Side-span interaction comes under the second of Professor Cross' classifications—action which produces participation stresses. It should be noted that maximum negative moment in the side span itself is primarily participation moment.

The preceding discussion on the change in sag is necessarily somewhat general because the three factors really merge into one another in a simultaneous action. Nevertheless, the proposed preliminary design method does give an idea of the related action of the various factors, a picture which is lost in mathematical formulas in the more exact methods.

The authors are to be congratulated upon their choice of subject and their manner of presentation. Considerable effort has apparently been made to keep the percentage error in the various approximations consistent, so that needless computation is avoided. The illustrative material has been carefully chosen to cover a wide range of cases. Obviously, the method may be extended for the computation of maximum moments at sections other than those given, should such extension be necessary or desirable in a given case.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

PRACTICAL METHODS OF RE-ZONING URBAN AREAS

Discussion

BY G. H. HERROLD, M. AM. SOC. C. E.

G. H. HERROLD,[10] M. AM. SOC. C. E. (by letter).[10a]—Much "water has gone over the dam" since the first zoning ordinance was drawn in 1916. Land has changed hands through the years. Lots have been purchased and many built upon in accordance with existing zoning. Skyscrapers have been constructed, filling stations established, apartments designed and built, new plats recorded, thoroughfares and parkways built, playgrounds and parks acquired; and a new conception of land tenure is under way.

To re-zone a city is quite an "upset" and should be approached cautiously and after much study. The average city in the United States that has grown naturally is divided into four classifications as to residential areas:

(1) There is the well-maintained and established high-class residential district in which properties are occupied by the owners. Each property is a specimen with an atmosphere created by the architecture and landscaping of the site and the culture of the owner;

(2) There is the neglected district lying between this stabilized residential district and the commercial or industrial district—neglected in the sense that no one is looking out for it collectively. The owners have lost faith in their investment and in the district;

(3) There is a completely blighted district or slum; and,

(4) There are the new outlying developments in which people in the stabilized districts are invited and importuned to invest.

The great good of zoning (that is, preserving the *status quo* in older districts and preventing the "mussing up" of new districts) has really been attained in

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NOTE.—The paper by Hugh E. Young, M. Am. Soc. C. E., was presented at the meeting of the City Planning Division, New York, N. Y., January 21, 1937, and published in March, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1938, by Messrs. Robert D. Kohn, Myron D. Downs, Harland Bartholomew, and Benjamin Saltzman.

[10] Member, State Planning Board; Office Engr., Dept. of Public Works; Planning Engr., The City Planning Board, St. Paul, Minn.

[10a] Received by the Secretary August 12, 1938.

the majority of cities. It is possible that the strengthening of health laws or a better enforcement of these laws in slum, blighted, or neglected districts may prove an aid to zoning. A slum district can be rehabilitated by the simple enforcement of cleanliness and structural safeguards, good municipal maintenance, and the enforcement of requirements for light and air. With this aid city planners may give the district a new lease on life under its present zoning.

All zoning ordinances provide for multiple-family dwellings. A multiple-family dwelling may be a barrack, a tenement building, a "four-plex," or a beautiful apartment building. Some definitions are necessary to segregate these different structures in zoning a city. If an apartment building was no higher than near-by residences and took the same setback; if it had an area of land, say, equivalent to 1 sq ft of open area to each square foot of floor area; if it had no common entranceways, it would be appropriate and acceptable in a residential district where houses are rented, especially if limited to locations, such as those adjoining along the side line of different zoning, facing a different zoning, or facing a park, school grounds, or institutional buildings, etc., the opportunities would be unlimited. An apartment building might be defined as a two and one-half story, multiple-family dwelling in which each apartment has its own individual entrance, etc. A tenement might be defined as a multiple-family dwelling with a central hall used in common by all tenants, etc.

Mr. Young shows that the great grievance is too much commercial, industrial, and apartment house zoning—a lack of balance in the proportioning of areas of zones and the placing of a value on such mis-zoned areas by the assessor that make it impossible to use it for homes.

In St. Paul, Minn., land and buildings are not assessed according to the zoning, they are assessed according to use—the zoning map being used as a guide. There is some unwarranted commercial strip zoning running through residential districts, but if residences predominate the land is assessed as residential property. This is the only rational method of assessing city property.

The only cure for poor administration, of course, is better administration. Zoning boards made up of a personnel with no planning experience are serious defects. Zoning is a part of the City Plan and can best be administered and interpreted by planning-minded people. In St. Paul, the Board of Zoning created by the Zoning Ordinance is the Zoning Committee (five members) of the City Planning Board; thus planning and zoning are tied together.

Some ordinances simply require the elimination of "strip" business zoning and the substitution of "district" business zoning to bring them into a fairly consistent situation as to proportioning the uses of property. This strip zoning, so common in existing ordinances, was brought about by members of "pressure groups," who either did not know what good zoning was or who wished to discredit zoning.

Some ordinances may be brought into line by provisions for terminating non-conforming uses after a period of years. This is an urgent need. In fact, the cure of the non-conforming evil and the reduction of footage of commercial zoning in residential developments might again carry us along over another 20-yr period.

In St. Paul, the Board of Zoning will recommend to the Council the remodeling of a non-conforming store building in a residence district, which is in distress financially, into a "triplex" or "four-plex" if it will move back to the building line of the homes and present a suitable architectural treatment.

In a city like Chicago, Ill., where changes come so rapidly, the proposed re-zoning based on a land-use survey and a proper proportioning of uses as determined by years of experience, is probably desirable, but a land-use survey must also be properly interpreted and this cannot be done by speculators.

Mr. Young has made a notable contribution to this subject of zoning and his analysis of the situation in Chicago, as well as the information gleaned through his questionnaire of December, 1936 (and which he has made available in his paper), will be of great value to all cities that are studying this matter of zoning.

The logical basis for re-zoning is the existing use of property and the anticipated normal absorption of additional property, not necessarily vacant property. This requires a sound knowledge of the trend of a city's growth. The causes of blight which were operating before zoning ordinances were devised and which were not cured by zoning must be rediscovered. Inadequate street, sewer, and water facilities, and poor maintenance were among the causes of blight. These defects must not be confused with alleged poor zoning. Suitable buffering must be considered. There must be a natural stepping up of uses between the district of no restrictions (industrial) and the district of highest restrictions (residence). The use of parkways for buffering industrial districts is probably one of the best.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

A THEORY OF SILT TRANSPORTATION

Discussion

BY MESSRS. G. W. HOWARD, HARRY F. BLANEY, AND E. W. LANE

G. W. HOWARD,[23] JUN. AM. SOC. C. E. (by letter).[23a]—Any stimulation of thought with regard to a subject as important as silt transportation is indeed welcome. The data presented as a result of the observations on the River Great Ouse and the Hundred Mile River are both interesting and enlightening.

In considering the application of the author's theory to river-control works, however, it is believed advisable that the term, "river," as used in this paper, be clarified. That the theory which is suitable for the Hundred Mile River may be applied to any other river is questionable. The Hundred Mile River, with its cross-sectional area of about 300 sq ft, is small when compared with the Lower Mississippi, which has a bank-full cross-sectional area of about 200 000 sq ft. The differences in hydraulic elements between the Hundred Mile River and most of the rivers in the United States are so great that this hypothesis of silt transportation should be used cautiously.

Of course, in the development of any theory, it is necessary to make certain basic assumptions. Some type of distribution of grain sizes is essential in formulating an equation. On the other hand, actual observations are of great value when one considers the characteristics of any stream. The remarks by Mr. H. S. Kahai[24] in connection with a silt survey of the channels at the Makhi Weir are pertinent. Mr. Kahai states:

"The following conclusions were drawn:

"(*A*) That it was not possible to determine a definite position on a vertical in any of the channels under observation to represent the mean velocity, silt charge, and weighted mean diameter for the cross-section."

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NOTE.—The paper by W. M. Griffith, Esq., was published in May, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1938, by Joe W. Johnson, Jun. Am. Soc. C. E.

[23] Junior Engr., U. S. Waterways Experiment Station, Vicksburg, Miss.

[23a] Received by the Secretary August 18, 1938.

[24] "Report on the Work Done in the Development and Research Division, Sind, for the Year Ending May, 1938," by H. S. Kahai, Executive Engr.

"(*C*) That the behavior of the quality of silt was so erratic throughout the cross-section that nothing could be said about its position of occurrence."

The only illustration in this report which referred to the study of silt, was a cross-section (approximately 750 sq ft in area) of the Mithrao Canal. The distribution for mean grain size and velocity in the canal were as shown in Fig. 10. The silt, however, had a mean grain diameter of 0.030 mm, which is smaller than that used in developing the author's theory, and falls into the class referred to as "light materials carried in suspension by all eddies." These results show the indefinite tendencies for the movement of silt. This also seems a logical conclusion for practically every stream of appreciable size that draws its silt load from tributaries and is of sufficient cross-sectional area not to be made-to-order.

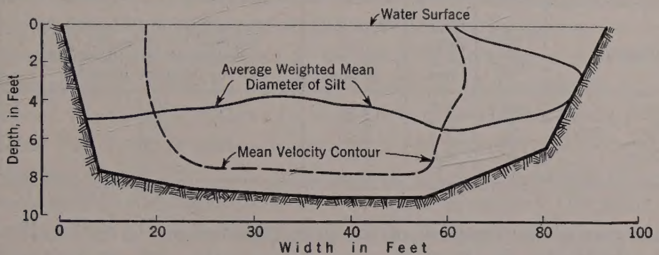

FIG. 10

Satisfactory formulas have been derived for various canals in certain localities, but the paucity of data from observations has not permitted the proper approach to the river problem. The magnitude of this problem can be seen from the recent study of variations of suspended silt along the Colorado River.[25] The publication of complete silt surveys and bed-load observations should enlighten the profession on the subject of larger streams to a greater degree within the next few years.

HARRY F. BLANEY,[26] M. AM. SOC. C. E. (by letter).[26a]—The theory of silt transportation advanced by Mr. Griffith is of special interest to engineers in the southwestern part of the United States where many of the rivers carry large quantities of silt. In this section one of the most important problems to be solved in designing irrigation and flood-control works properly is that of estimating the silt load carried by the stream. In most instances long-period records of silt measurements are not available, and estimates are based on fragmentary or short-period records. However, long-period stream-flow records may be available and, consequently, the author's method would be valuable in estimating silt loads.

It is noted that Mr. Griffith uses some of the silt studies made by the writer in Imperial Valley, California, and reported[10] in 1928, to illustrate his theory of silt transportation. In reviewing his original field notes the writer has found some unpublished observational data and has used these to check Equation (13). The results are shown in Table 8.

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[25] *Transactions*, Am. Soc. C. E., Vol. 94 (1930), pp. 1104–1151.

[26] Irrig. Engr., U. S. Bureau of Agricultural Eng., Los Angeles, Calif.

[26a] Received by the Secretary September 2, 1938.

[10] "Silt in the Colorado River and Its Relation to Irrigation," *Technical Bulletin No. 67* (1928), U. S. Dept. of Agriculture.

TABLE 8.—ANALYSIS OF SILT DISTRIBUTION IN NO. 5 MAIN CANAL AND EAST HIGHLINE CANAL, IMPERIAL VALLEY, CALIFORNIA

(Observations made on or near center vertical of cross-section)

Name of canal	Width, in feet	Total depth, d, in feet	Velocity, v, in feet per second	Proportion of silt by weight retained by Sieve No. 200, expressed in parts per 10 000 (arithmetic mean)	Values of c_1 in Equation (13)
No. 5 Main	32	3.3	3.48	10.3	13.2
No. 5 Main	25	3.9	3.79	18.7	13.1
East Highline	40	4.6	2.85	8.5	8.9
East Highline	40	4.6	1.92	6.9	6.0

Perhaps some of the results[27] of the investigation made by Orville A. Faris, M. Am. Soc. C. E., for the U. S. Bureau of Agricultural Engineering, in Texas, could be used to check Mr. Griffith. The writer does not have access to the original data on Texas streams but, in one instance, for observations on the Brazos River, the computed value for $\frac{7.5\, v}{d^{0.57}}$ amounts to 7.9 as compared with observed value, 9.6 parts per 10 000. However, in another instance, the agreement is not so close.

E. W. LANE,[28] M. AM. SOC. C. E. (by letter).[28a]—The author's theory regarding the shape of a channel in erodible material is based on the hypothesis that the fundamental relation between depth and velocity is of the type:

$$B = c\, d^n \qquad (15)$$

There can be little doubt that some such rough relation holds as is shown by the many results obtained by observing stable channels under a wide range of conditions.[29] However, this relation is believed to be an empirical one which gives only approximate results.

The author has used the Chezy equation which has long been known to be unreliable when applied to open channels. If the more reliable Manning's relation is assumed, Mr. Griffith's reasoning indicates that the value of n should be close to 0.67 rather than 0.50.

Mr. Griffith believes that the size of the material of which the banks and bed of a channel are composed does not affect the shape that they will assume, as long as they are of granular material of a fineness between 0.002 in. and shingle. The writer does not believe that any formula can be found covering such a wide range of materials unless the material size is included as a factor. The writer[29] has expressed the belief that the size of the material forming the bed and both the quantity and the size of the load carried by the stream were factors in determining the channel shape. In that paper, emphasis was placed on the importance of the quantity of the material carried since the Lacey formula (which was then under active discussion in India) neglected that factor. Mr. Griffith's relations consider the quantity of material carried, but within the aforementioned very wide range of sizes, ignore the factor of material size and the writer wishes again to urge that both size and quantity are essential factors.

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[27] "The Silt Load of Texas Streams," *Technical Bulletin No. 382* (1933), U. S. Dept. of Agriculture.

[28] Prof., Hydr. Eng., Univ. of Iowa, Iowa City, Iowa.

[28a] Received by the Secretary September 16, 1938.

[29] "Stable Channels in Erodible Material," by E. W. Lane, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 126.

The resistance of granular material to motion by flowing water is closely related to the velocity of settlement in water of the particles of which it is composed. The settling velocity of a particle 0.002 in. in diameter is approximately 0.2 cm per sec, whereas particles 1 in. in diameter would fall about 50 cm per sec, or more than 250 times as fast, and the particles composing a shingle bed might be several times larger than 1 in. in diameter. It does not seem reasonable that beds of such widely different material would act in exactly the same manner.

The writer does not believe that any formula can be found, even for a single stream, that will give the relation between the velocity and depth in any vertical and the total silt load at that point of a size greater than 0.002 in. It is well known that most streams carry widely different suspended loads at the same discharge. For example, at one time, a stream may carry a given load at a certain discharge; and, at another time, it may carry twenty times as much with the same flow.

The quantity of silt carried in an ordinary river channel is composed of two parts, one of which is of the sizes found in the bed, which the river is constantly picking up from the bed and which are also constantly settling to the bottom again. The second part is composed of material brought in by the tributaries which is so fine that it is kept in suspension almost continuously. The river is usually able to carry many times the load of this fine material which the tributaries bring to it. The quantity of this material in suspension at any time, therefore, is usually independent of the nature of the main channel, and depends solely on what the tributaries provide.

For a given river, there is a definite relation between the first part of the load as described herein and the discharge, but since the total load is composed of both parts, and since the second part bears no relation to the discharge, the sum of the two is almost independent of the discharge. If the lower limit of the size of bed material were 0.002 in., the material carried of a size greater than that, being all obtainable from the bed, would have a definite relation to the discharge. However, in most rivers having granular beds, the minimum size of bed material is about 0.005 in. and a large part of the suspended load lies between this size and 0.002 in. In most rivers, therefore, the quantity of this material bears no relation to the discharge or velocity, and the total greater than 0.002 in., since it includes this range, would also bear no relation to them.

In the present state of the knowledge of silt transportation, with the tremendous range of silt concentrations encountered and the inaccuracies that probably exist even in the best silt measurements, differences between observed and computed results which would be intolerable in other branches of engineering science, in this field, may represent important contributions to that science. The relatively excellent agreement of the observed data with the values computed by Mr. Griffith's formulas, however, is believed to be fortuitous.

For example, in Tables 1 and 2 the data are expressed in proportions of silt by weight retained on the 200-(0.0029-in.) mesh sieve, but in Table 3 the percentage retained on the 300-(0.0018-in.) mesh sieve was used. This is the way the data were presented in the original source. A computation from the original data on which Table 3 is based, however, shows that if the 200-mesh sieve had been used as the limit, the results would have been only 60% as great. This indicates that if the division point of Tables 1 and 2 had been at 0.002 in. instead of 0.0029 in., the results would have been roughly 55% larger than those given. Data published elsewhere in the *Bulletin* from which Tables 1, 2, and 3 were taken[10] do not substantiate Mr. Griffith's results. Using the data for the Elder Canal (Table 23),[10] results are obtained giving silt concentrations averaging only 4% of the values given by Mr. Griffith's formula, and, for the Rositas Canal,[10] results averaging only 38 per cent.

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[10] *Technical Bulletin No. 67* (1928), U. S. Dept. of Agriculture.